Tutorial 8 – Minimal *versus* classical

SUMMER SEMESTER 2026 — FLORENT SCHAFFHAUSER

Due by July 6th at 4 PM.

We consider the natural deduction rules for well-formed formulas constructed in the lectures. Specifically, we consider the introduction and elimination rules of *minimal logic* and we recall that the well-formed formula `¬P` is defined as `P ⇒ ⊥`.

We now want to characterise classical logic in terms of minimal logic plus some axioms. Basically, we want to show something like the following:

"classical logic = minimal logic + `dne` = minimal logic + `contrapose`" .

QUESTION 1. State the double elimination rule in the form of a proof tree (call it `dne`).

QUESTION 2. In Rocq, the double elimination rule can be written as `dne : ∀ {P : Prop}, ¬¬P → P`. Construct a type called `MySpec` such that the following program is of type `MySpec` (in Rocq).

```
Theorem via_reductio_ad_absurdum {A B : Prop} (H : (B ∧ ¬A) → False) : MySpec.
  Proof.
    intro var1.
    apply dne.
    intro var2.
    apply H.
    split.
    - exact var1.
    - exact var2.
  Qed.
```

QUESTION 3. Consider the following natural deduction rule.

$$\frac{\Gamma \vdash \neg B \Rightarrow \neg A}{\Gamma \vdash A \Rightarrow B}\,\texttt{contrapose}$$

Read this rule from bottom to top and explain it in in words. How would you write a rule like that in Rocq?

QUESTION 4. Show, using only minimal logic and `dne`, that the `contrapose` rule holds in calssical logic (write the derivation as a proof tree).

QUESTION 5. As a converse to the previous question, show, using only the rules of minimal logic, that if `contrapose` holds, then `dne` holds.