

Tutorial 6 – Natural deduction

SUMMER SEMESTER 2026

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Due by June 29th at 4 PM.

EXERCISE 1. Let A , B and C be well-formed formulas and let Γ be a context. Show that the following sequents hold in minimal logic.

1. $\Gamma \vdash_m (A \Rightarrow B \Rightarrow C) \Leftrightarrow ((A \wedge B) \Rightarrow C)$
2. $\Gamma \vdash_m (C \Rightarrow A) \vee (C \Rightarrow B) \Rightarrow (C \Rightarrow A \vee B)$
3. $\Gamma \vdash_m (A \Rightarrow B) \Rightarrow (\neg B \Rightarrow \neg A)$
4. $\Gamma \vdash_m \neg(A \vee B) \Leftrightarrow (\neg A \wedge \neg B)$
5. $\Gamma \vdash_m (\neg A \vee \neg B) \Rightarrow \neg(A \wedge B)$
6. $\Gamma \vdash_m A \Rightarrow \neg\neg A$
7. $\Gamma \vdash_m \neg B \Leftrightarrow \neg\neg\neg B$
8. $\Gamma \vdash_m \neg\neg(A \vee \neg A)$

You can choose which proofs to present as a proof tree. For instance, if you have an equivalence to prove, you can choose to write a proof tree for each implication.

EXERCISE 2. Let A , B and C be well-formed formulas and let Γ be a context. Show that the following sequents hold in classical logic.

1. $\Gamma \vdash_c (\neg B \Rightarrow \neg A) \Rightarrow (A \Rightarrow B)$
2. (**Peirce's law**) $\Gamma \vdash_c ((A \Rightarrow B) \Rightarrow A) \Rightarrow A$
3. $\Gamma \vdash_c (\neg A \vee \neg B) \Leftrightarrow \neg(A \wedge B)$

EXERCISE 3. Recall that Boolean semantics provide a sound interpretation of minimal logic. Explain why they also provide also a sound interpretation of intuitionistic and classical logic.

EXERCISE 4. Recall that a well-formed formula F is called a *tautology* if the entailment $\vDash F$ holds (in Boolean semantics). Similarly, call F an *absurdity* if $\vDash \neg F$ holds. Now assume that F is an absurdity and show that, for all well-formed formula G , the entailments $\vDash F \Rightarrow G$ and $F \vDash G$ hold.