

Tutorial 3 – Boolean semantics

SUMMER SEMESTER 2026

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Due by May 26th at 4 PM.

EXERCISE 1. (A relation between validity and satisfiability) Let F be a well-formed formula in the sense of the lectures. Show that F is valid if and only if $\neg F$ is not satisfiable.

Note: Refrain from using symbols to abbreviate concepts such as universal and existential quantification and make sure to explain your reasoning in detail.

EXERCISE 2. (Expressing semantic equivalence in terms of a valid formula) Let F and F' be well-formed formulas. Show that F and F' are semantically equivalent if and only if $F \Leftrightarrow F'$ is a tautology.

Note: Again, really spell out and streamline your reasoning. Be sure to work on a draft first, to really understand what is going on before writing a clean copy.

EXERCISE 3. (Some tautologies in Boolean semantics) Let F and G be well-formed formulas. Show that the following formulas are valid from the point of view of Boolean semantics:

a. $F \vee \neg F$

c. $F \Leftrightarrow \neg\neg F$

b. $\neg(F \wedge \neg F)$

d. $F \wedge \neg F \Rightarrow G$

EXERCISE 4. (Semantic equivalence)

Let F and F' be well-formed formulas. Show the following semantic equivalences (for Boolean semantics).

a. $F \Rightarrow F' \equiv \neg F \vee F'$

c. $\neg(F \wedge F') \equiv \neg F \vee \neg F'$

b. $F \Rightarrow F' \equiv \neg(F \wedge \neg F')$

d. $\neg(F \vee F') \equiv \neg F \wedge \neg F'$

EXERCISE 5. (Congruence properties) Let F and F' be well-formed formulas. Show the following properties.

a. If $F \equiv F'$, then $\neg F \equiv \neg F'$.

b. If $F \equiv F'$, then for every well-formed formula F'' and every binary constructor $\diamond \in \{\wedge, \vee, \Rightarrow, \Leftrightarrow\}$, one has $F \diamond F'' \equiv F' \diamond F''$.