Right angles and reflections through a line

Franziska Thiele

Topics

The right angle

Perpendiculars

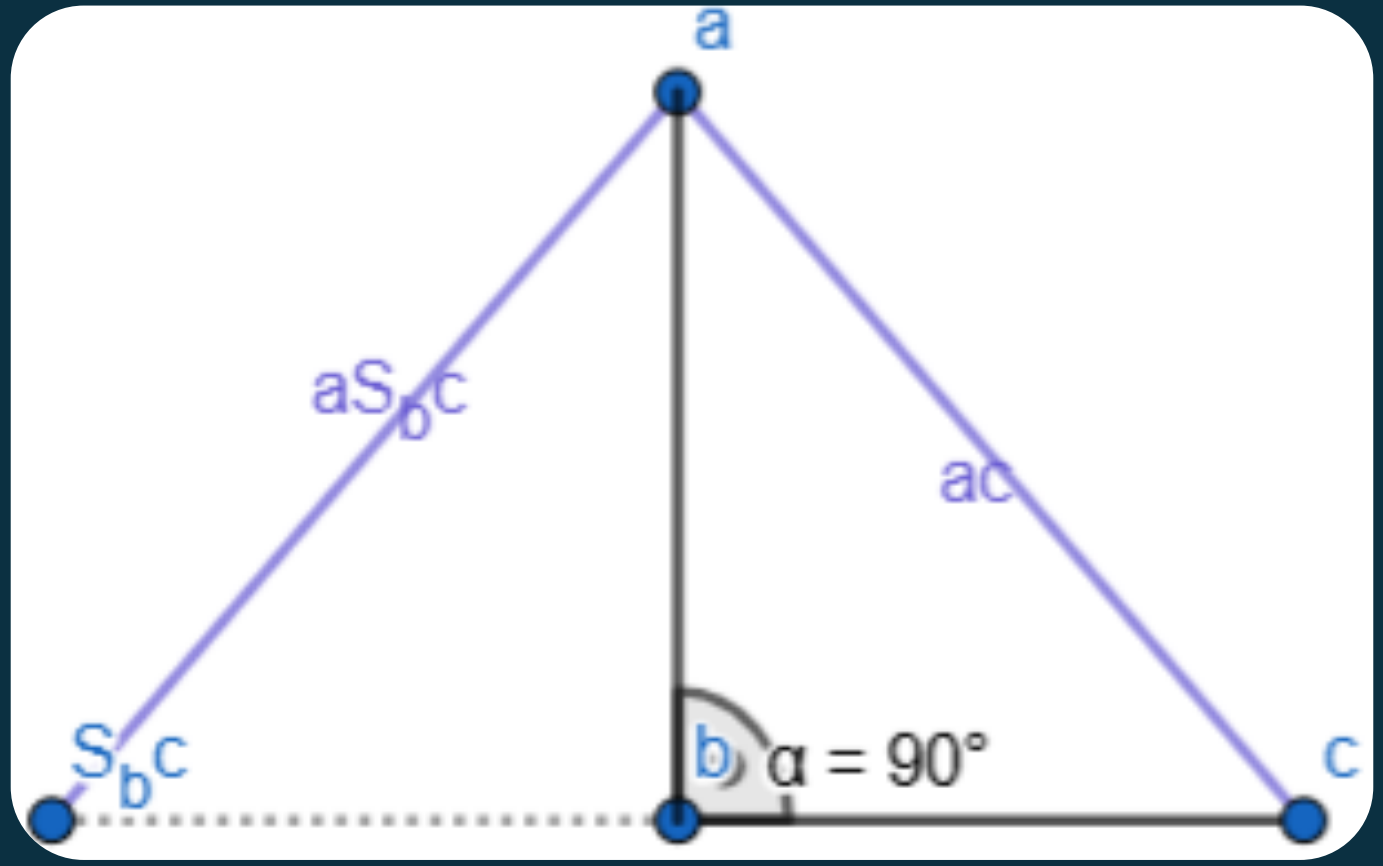
Middle Point

Reflection of a point through a line

Proof of theorem 10.10

8.1 Right angle

$$Rabc \iff ac \equiv aS_b(c)$$



8.2 Symmetry of right angles

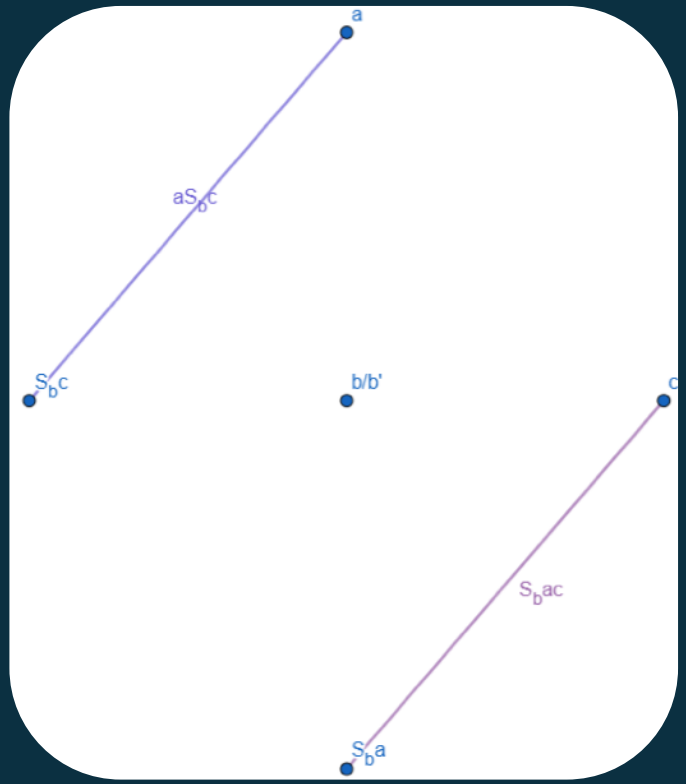
$$Rabc \rightarrow Rcba \iff ca \equiv cS_b(a)$$

$$\rightarrow \mathrm{ac} \equiv \mathrm{aS_b(c)} \leftrightarrow \mathrm{ca} \equiv \mathrm{cS_b(a)}$$

8.2 Symmetry of right angles

$$Rabc \rightarrow Rcba \iff ca \equiv cS_b(a)$$

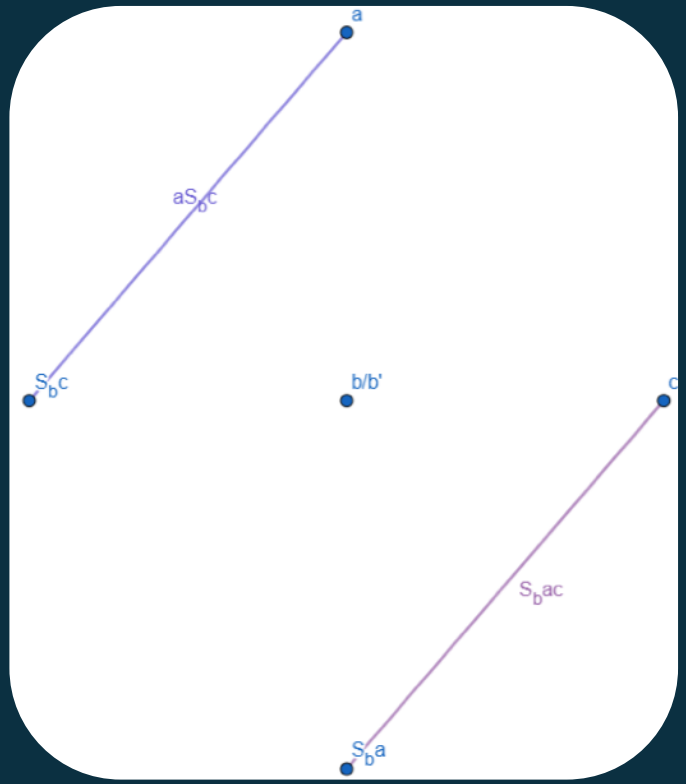
With 7.13: $aS_b(c) \equiv S_b(a)c$



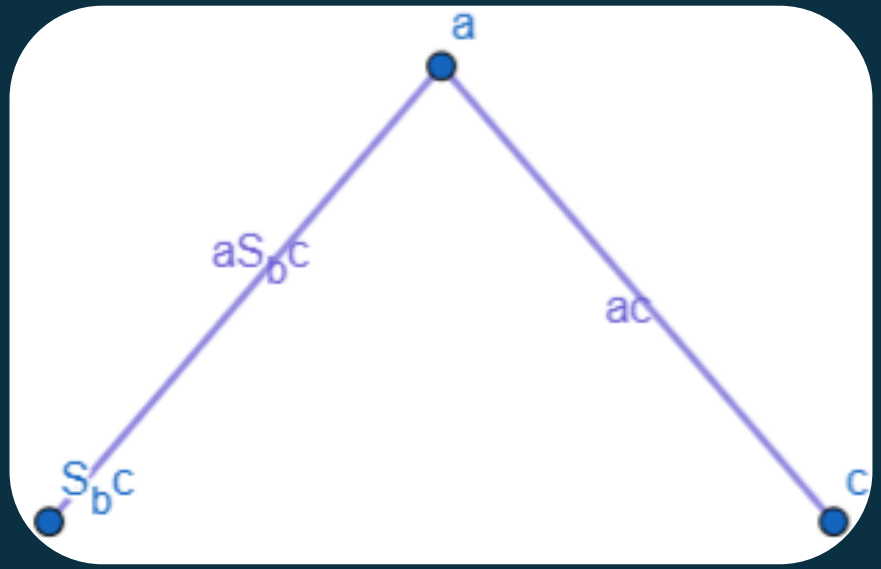
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With 8.1: $ac \equiv aS_b(c)$



8.2 Symmetry of right angles

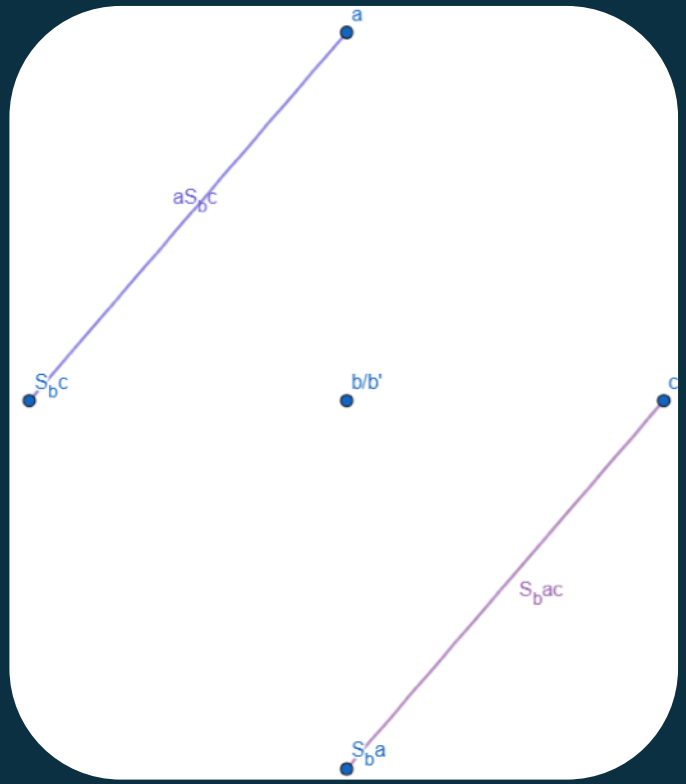
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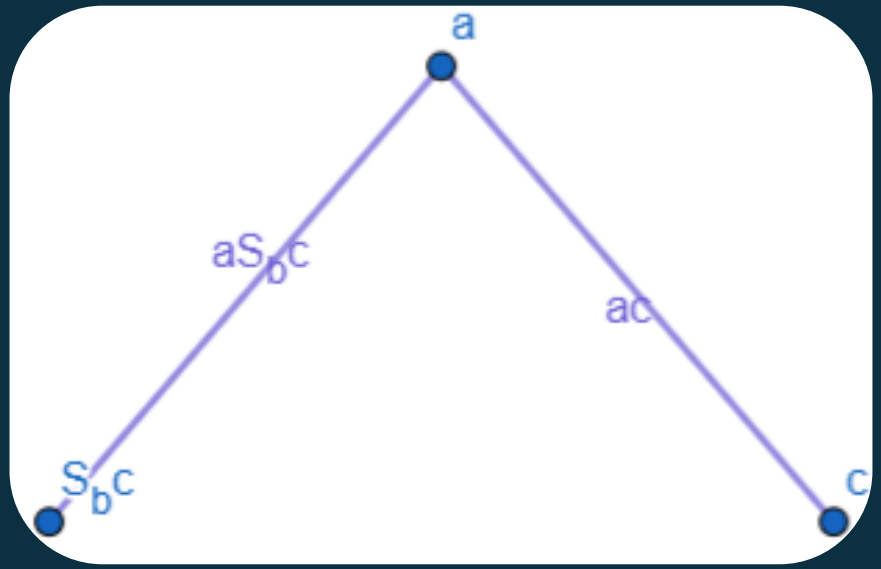
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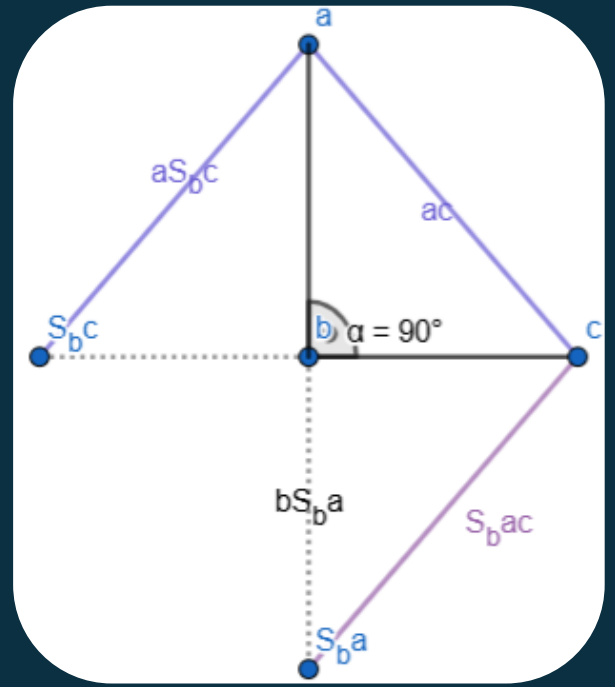
With 8.1: $ac \equiv aS_b(c)$

$$ca \overset{A1}{\equiv} ac \overset{8.1}{\equiv} aS_b(c) \overset{7.13}{\equiv} S_b(a)c \overset{A1}{\equiv} cS_b(a)$$

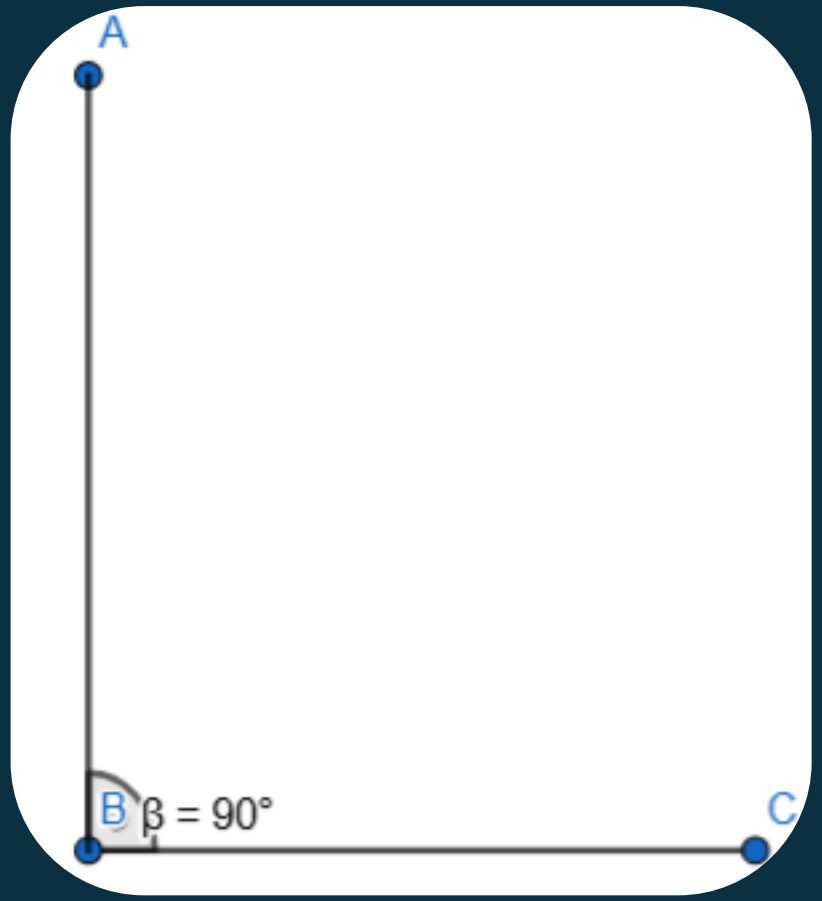
$\leftrightarrow$ Rcba



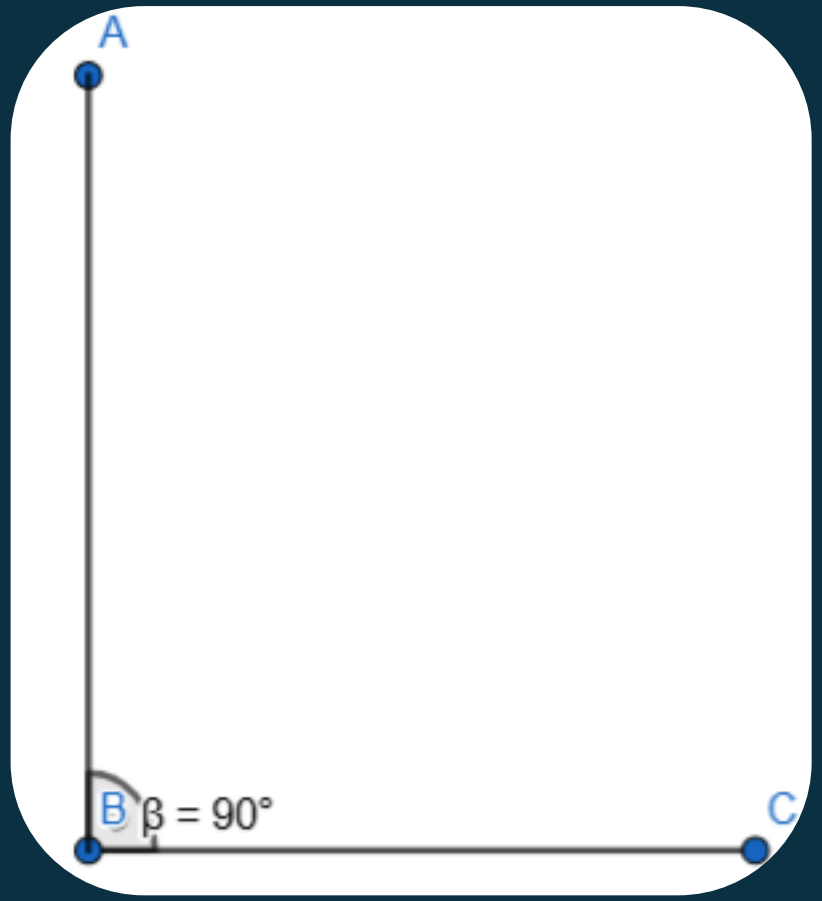


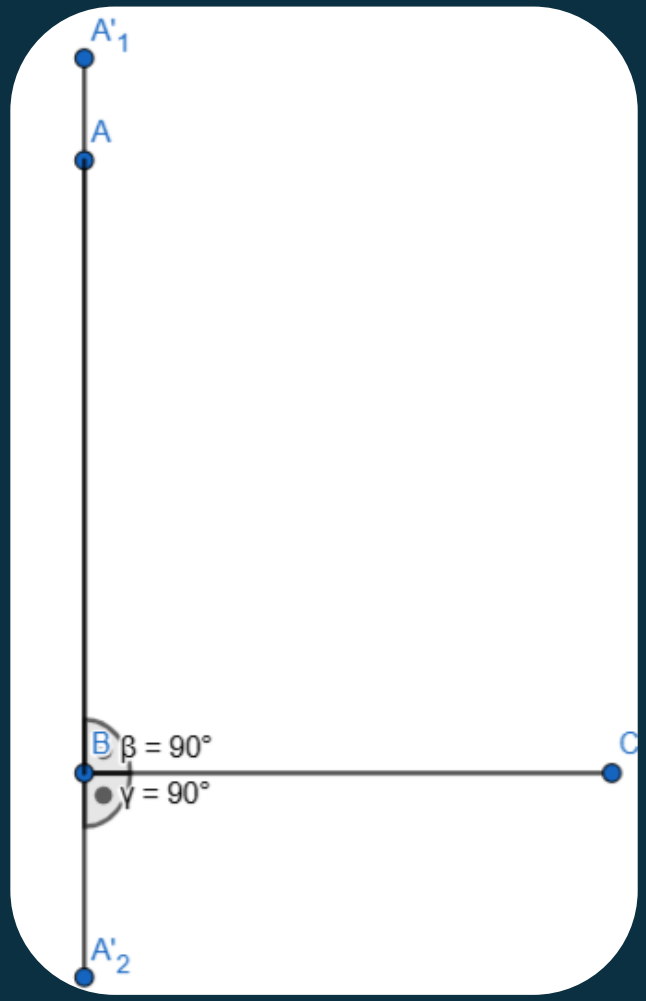


8.3 . $Rabc \wedge a \neq b \wedge Colbaa' \rightarrow Ra'bc$



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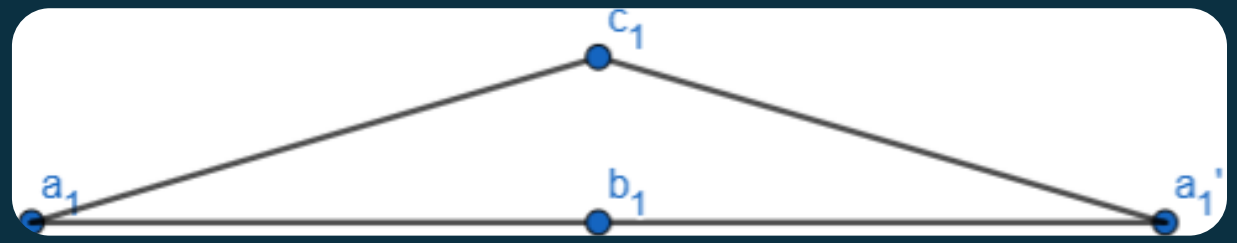
Right angle between two points

8.5. $\mathrm{Rabb} \leftrightarrow \mathrm{ab} \equiv \mathrm{aS_b(b)}$

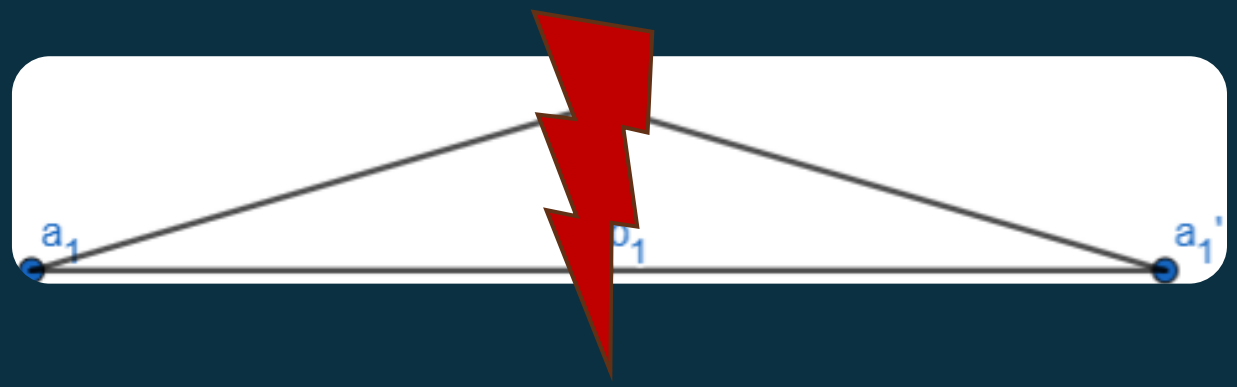


8.6 . $Rabc \wedge Ra'bc \wedge Baca' \rightarrow$

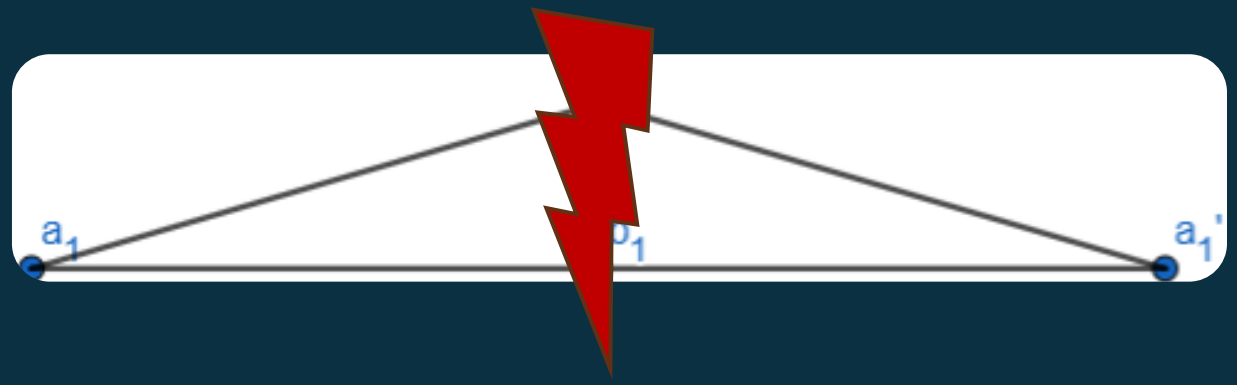
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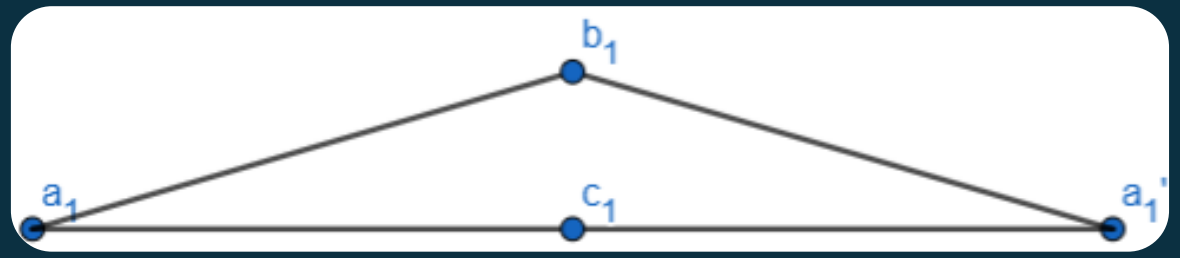


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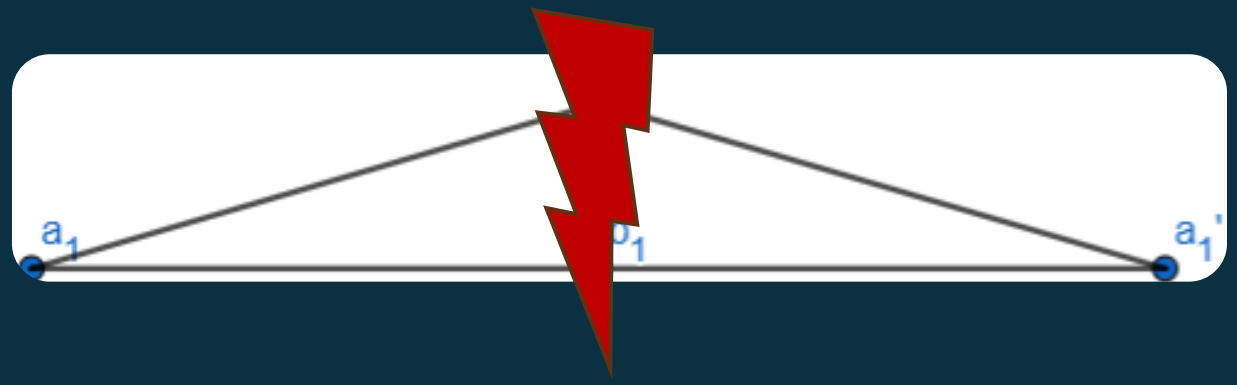


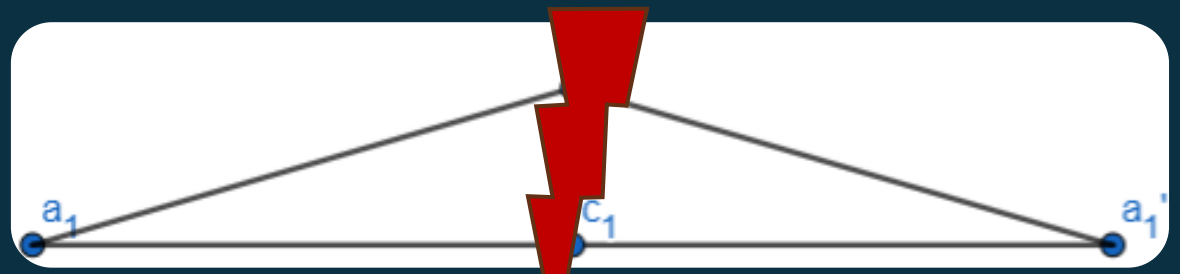
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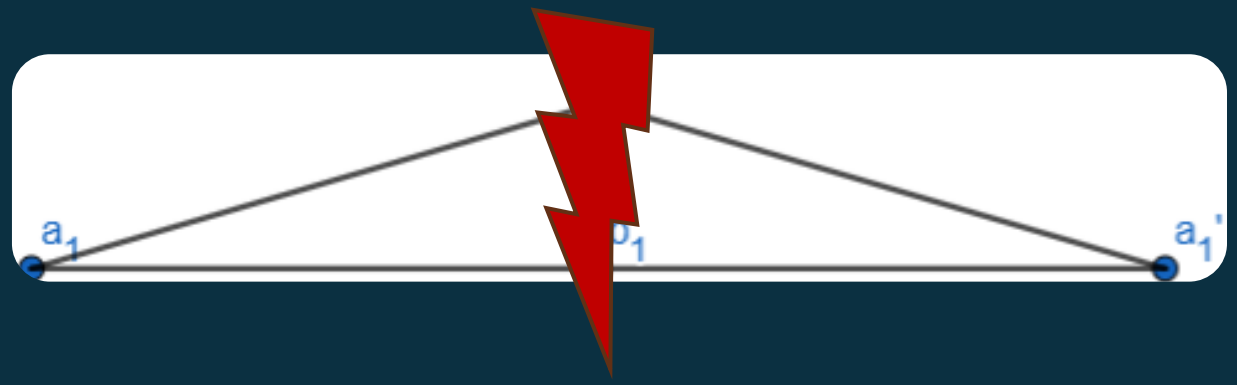


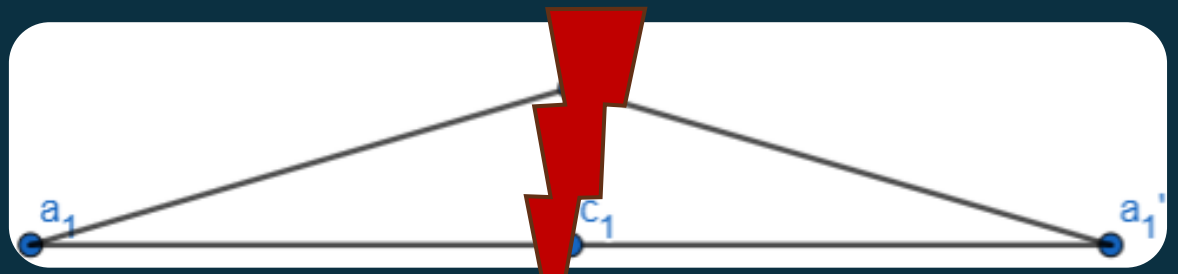
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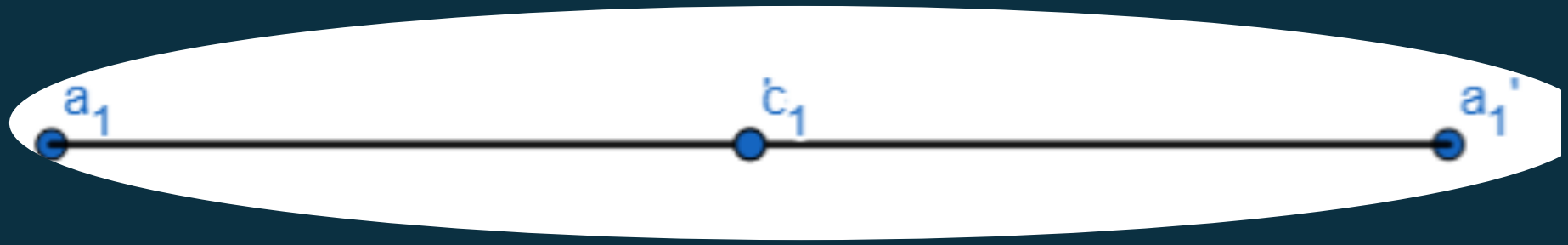




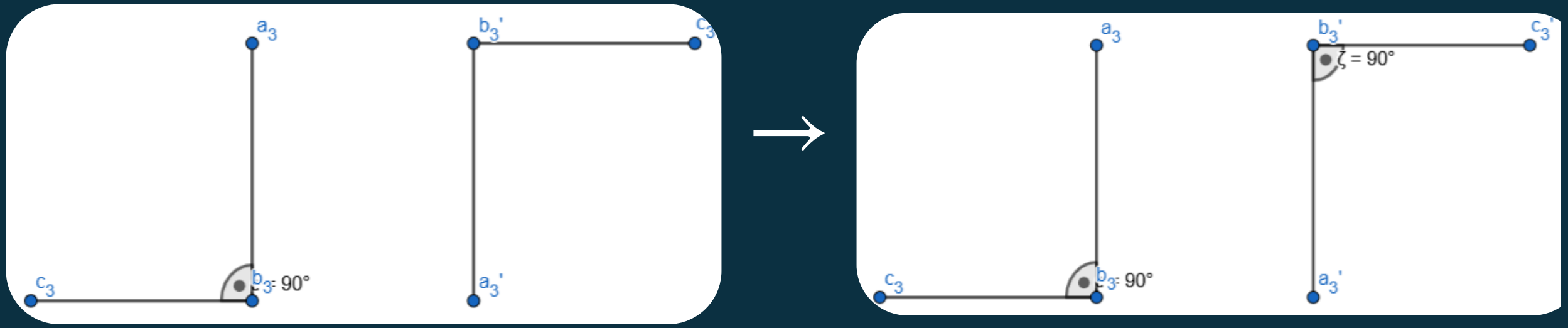
8.6 . $Rabc \wedge Ra'bc \wedge Baca' \rightarrow b = c$



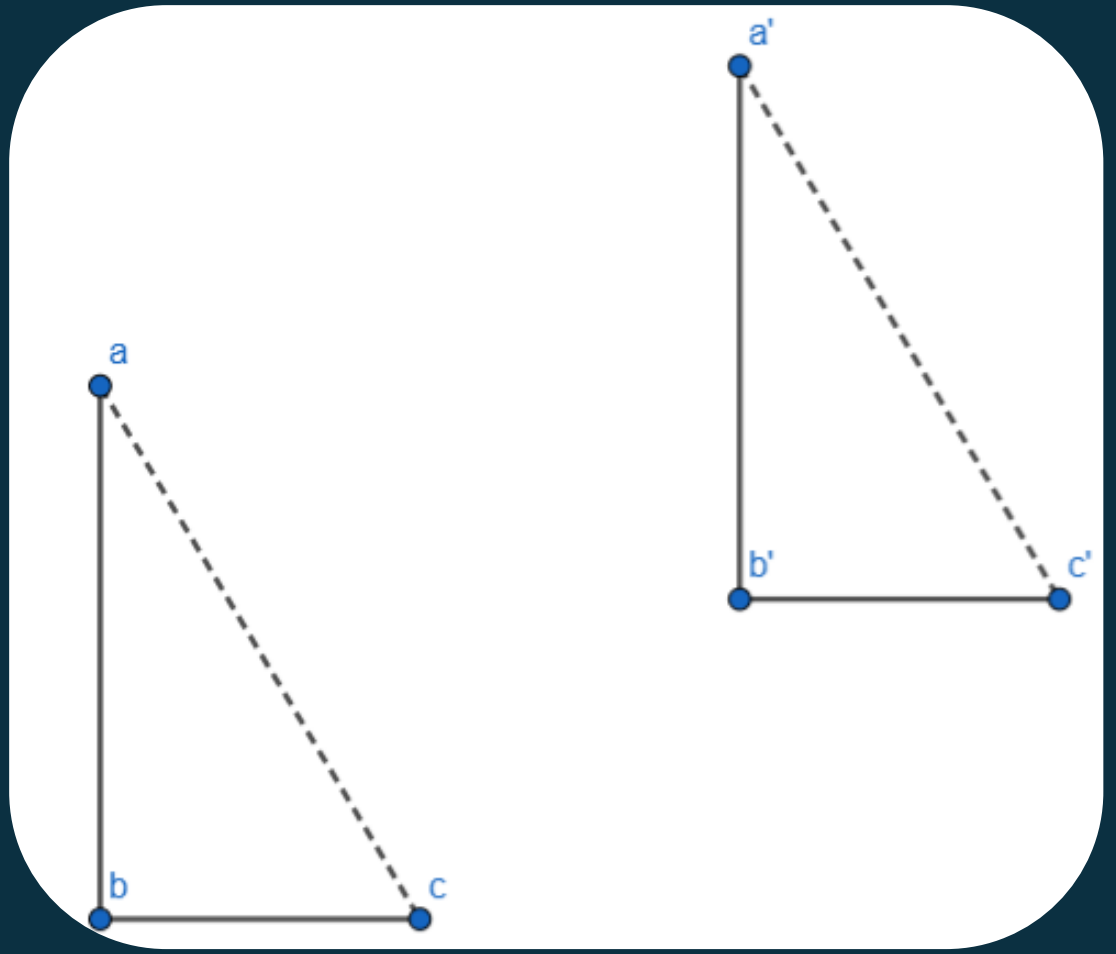




8.10 . $Rabc \wedge (abc) \equiv (a'b'c') \rightarrow Ra'b'c'$

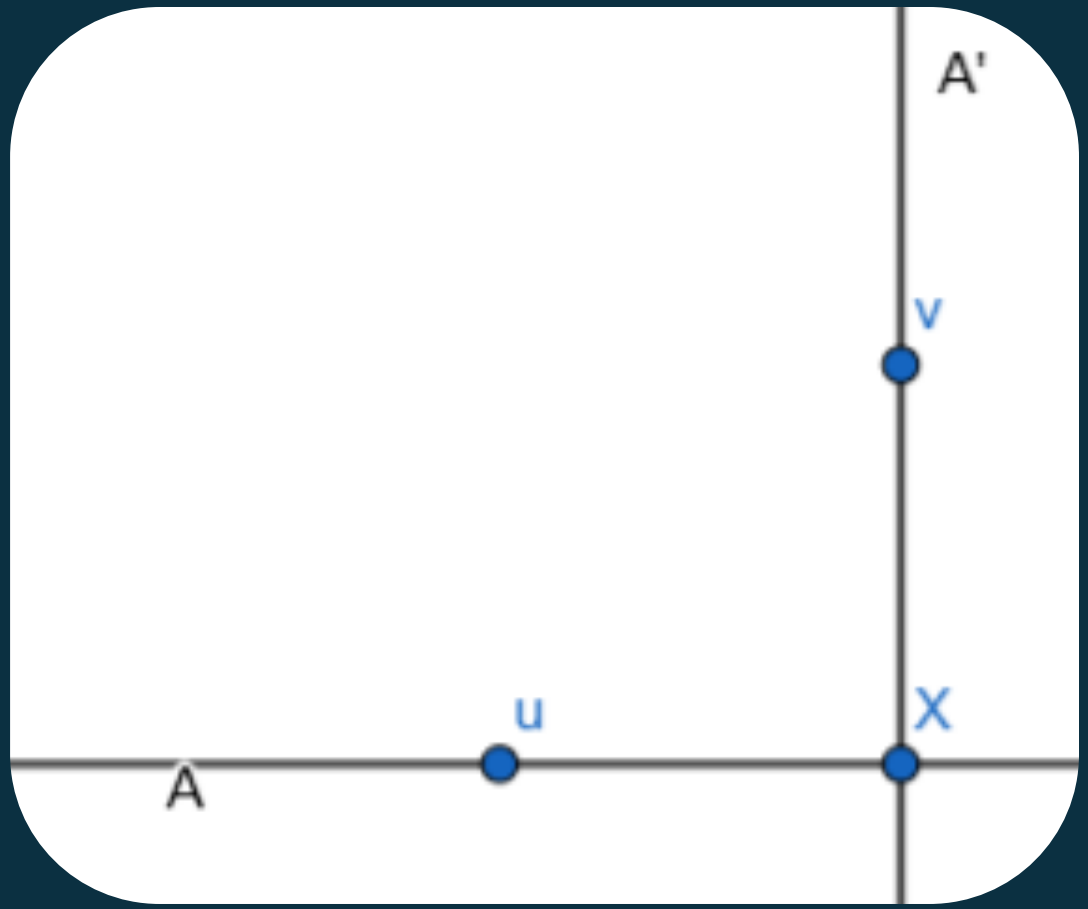


10.12 . $Rabc \wedge Ra'b'c'$
$\wedge ab \equiv a'b'$
$\wedge bc \equiv b'c'$
$\rightarrow ac \equiv a'c'$



8.11 Right angles between lines

$$A \perp_x A' :\Longleftrightarrow LnA \wedge LnA'$$
$$\wedge\, x \in A \wedge x \in A'$$
$$\wedge\, \forall u \forall v : (u \in A \wedge v \in A' \rightarrow Ruxv)$$

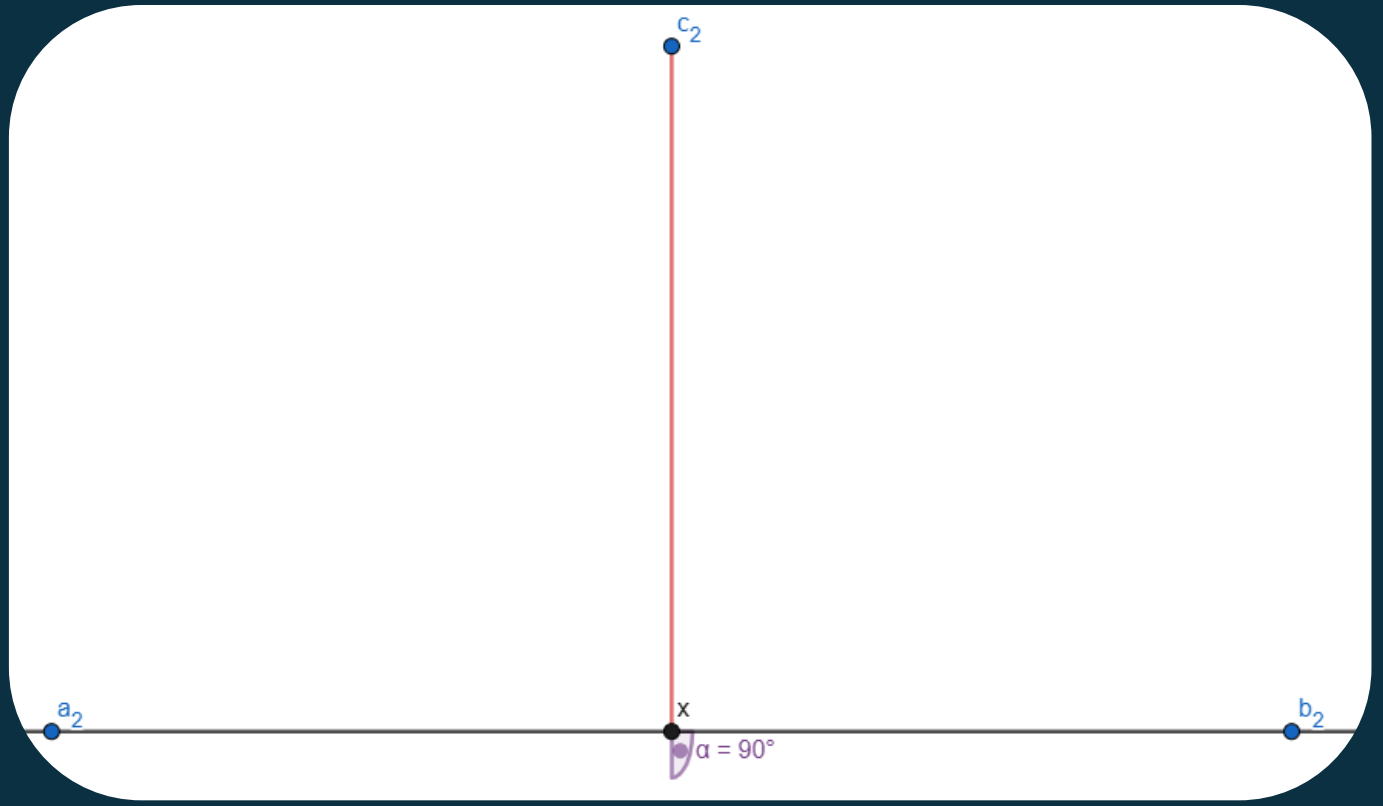


Perpendicular

8.17 is $\mathrm{ab} \perp \mathrm{cx} \wedge \mathrm{Colabx}$ then we call the line L(cx) a perpendicular in x to the line L(ab)

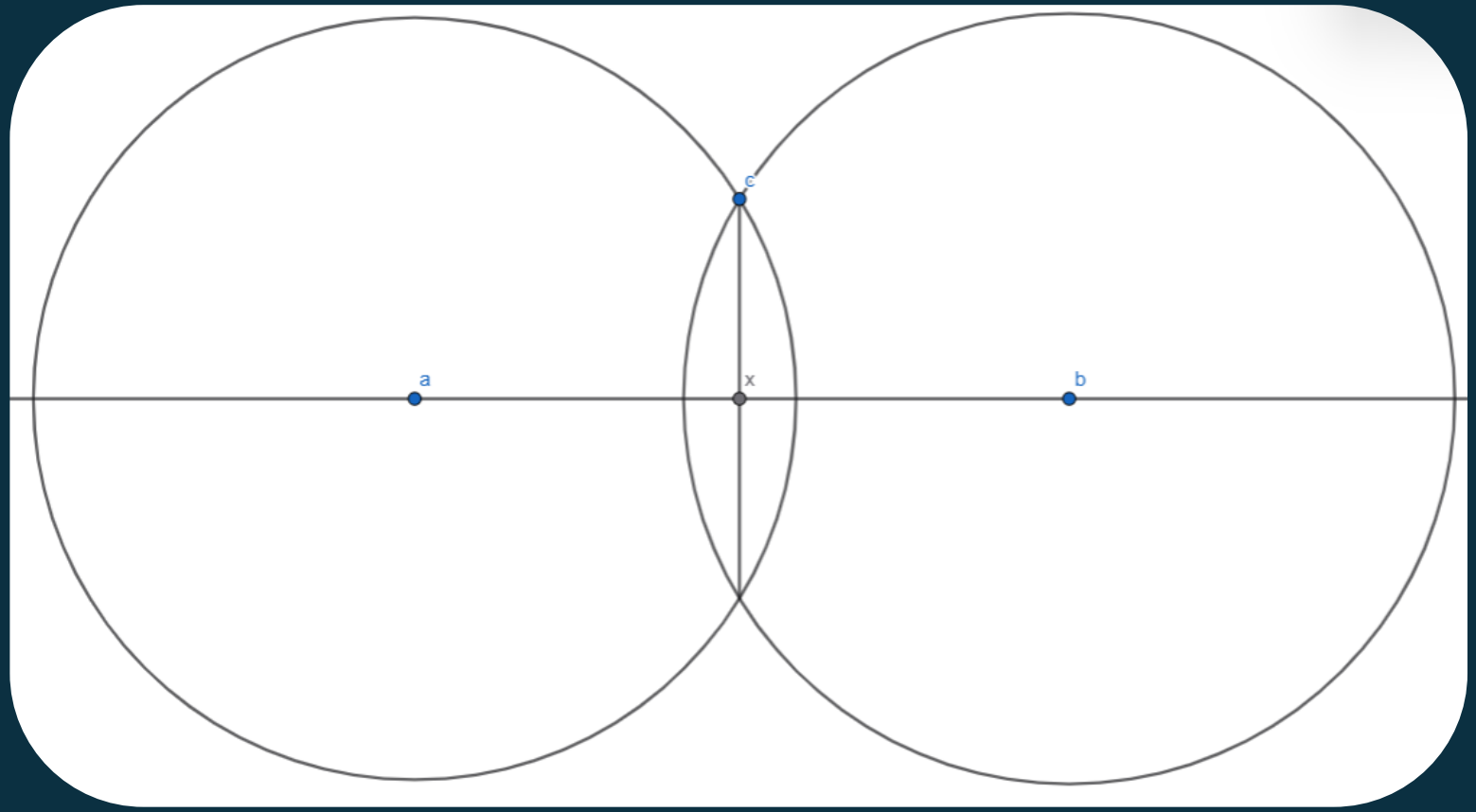
x is called the foot

of perpendicular



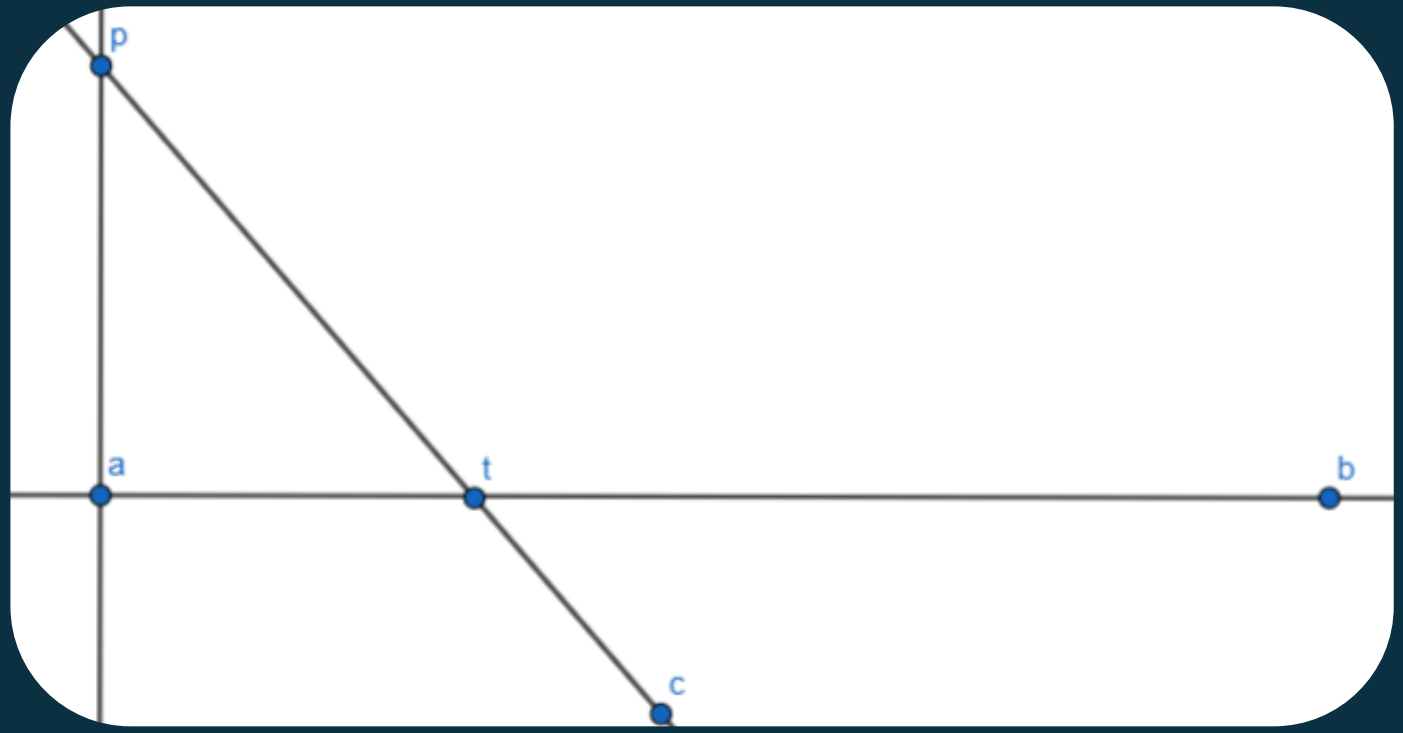
Theorem of the perpendicular

$$8.18\,. \quad \neg Colabc \rightarrow \exists! x \left[Colabx \wedge ab \perp cx \right]$$



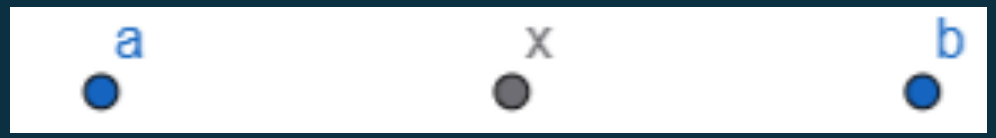
8.21

$$a \neq b \rightarrow \exists p \exists t \left[ab \perp pa \wedge Colabt \wedge Bctp \right]$$



10.1 Middle point

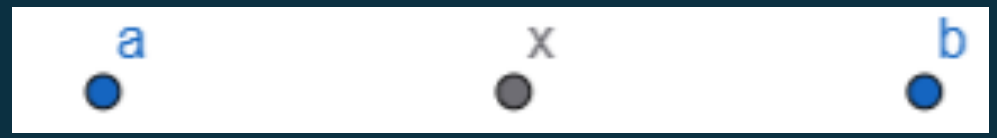
$$M(ab) := xMaxb$$



10.1 Middle point

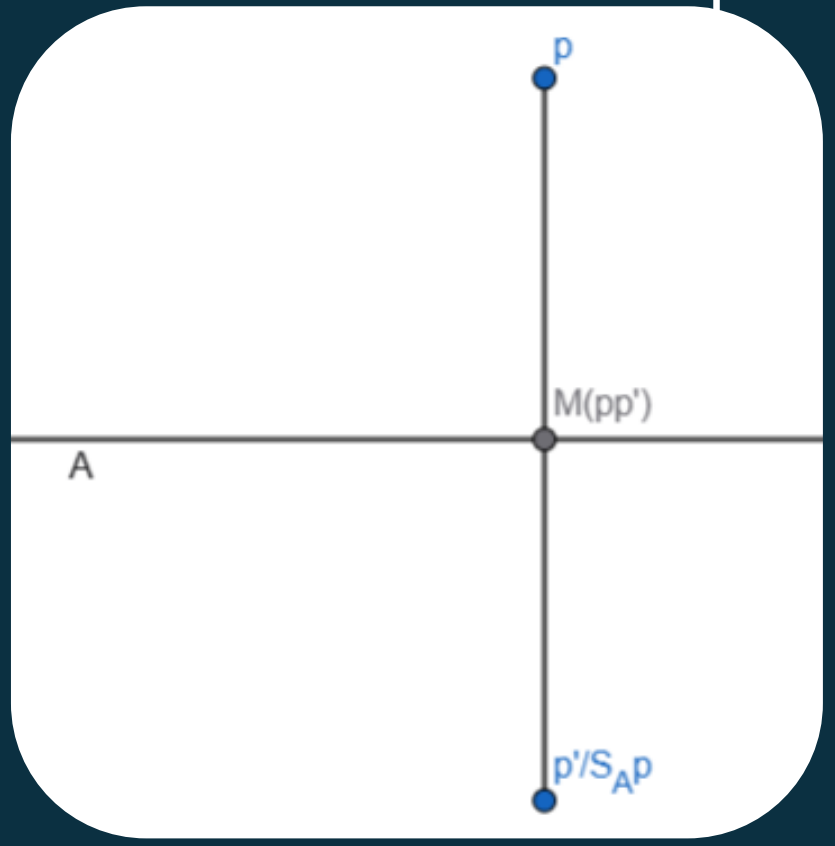
$$M(ab) :=: xMaxb$$

8.22 . $\forall a \forall b \exists! x \, Maxb$



10.2 Middle points on a perpendicular line

$$LnA \rightarrow \forall p \exists! p' \left[M(pp') \in A \wedge \left(A \perp pp' \vee p = p' \right) \right]$$



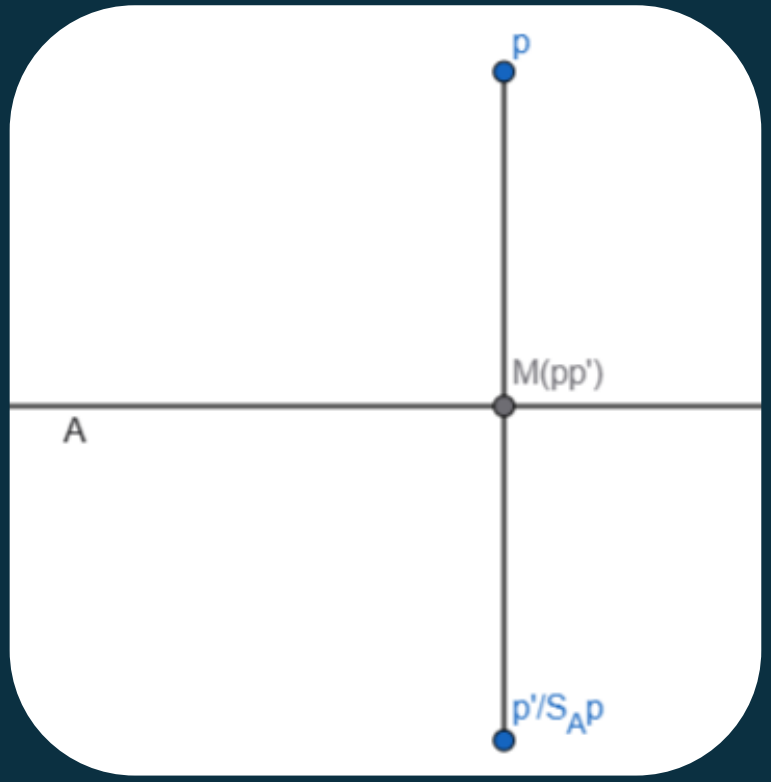


10.3 Reflection of a point through a line

$$S_A p = S_A(p) := p' \left[M(pp') \in A \wedge \left(A \perp pp' \vee p = p' \right) \right]$$

$$S_{ab} := \begin{cases} S_{L(ab)} & \text{if } a \neq b \\ S_a & \text{if } a = b \end{cases}$$



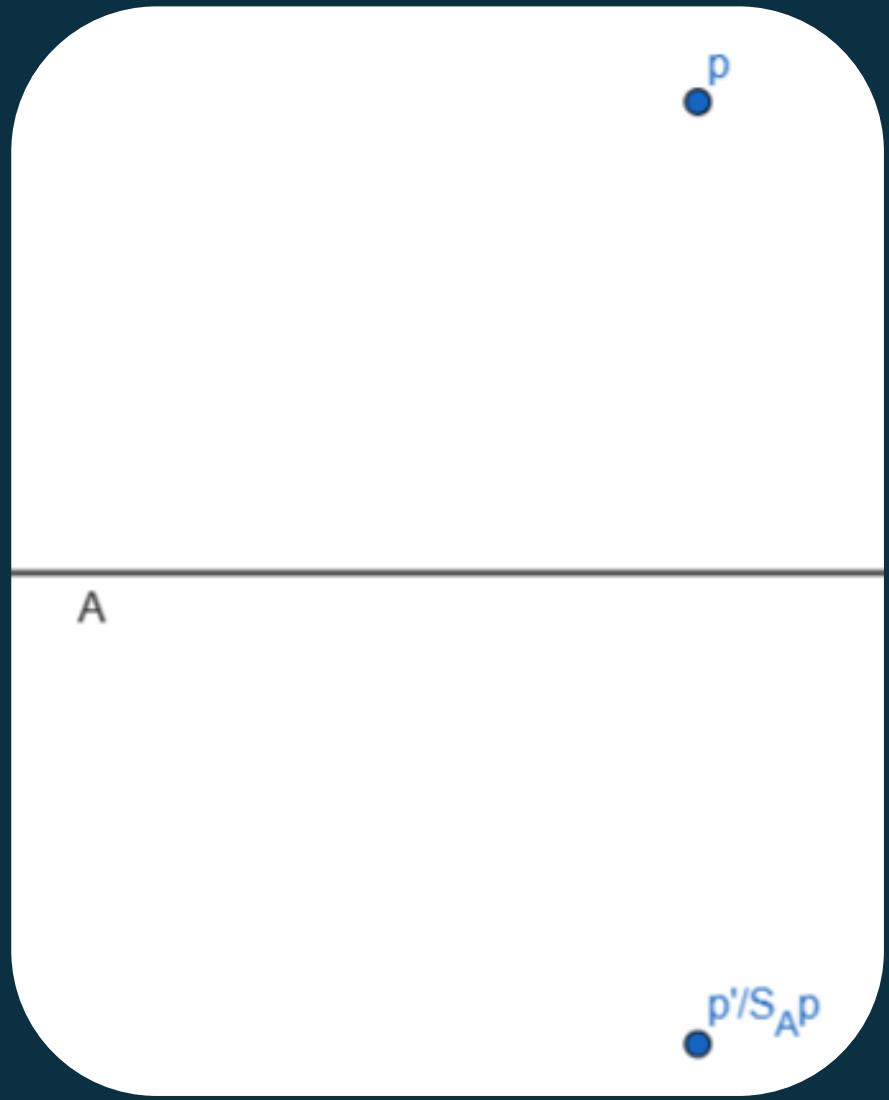


Conclusions of 10.3

10.4 . $S_A p = p' \iff S_A p' = p$

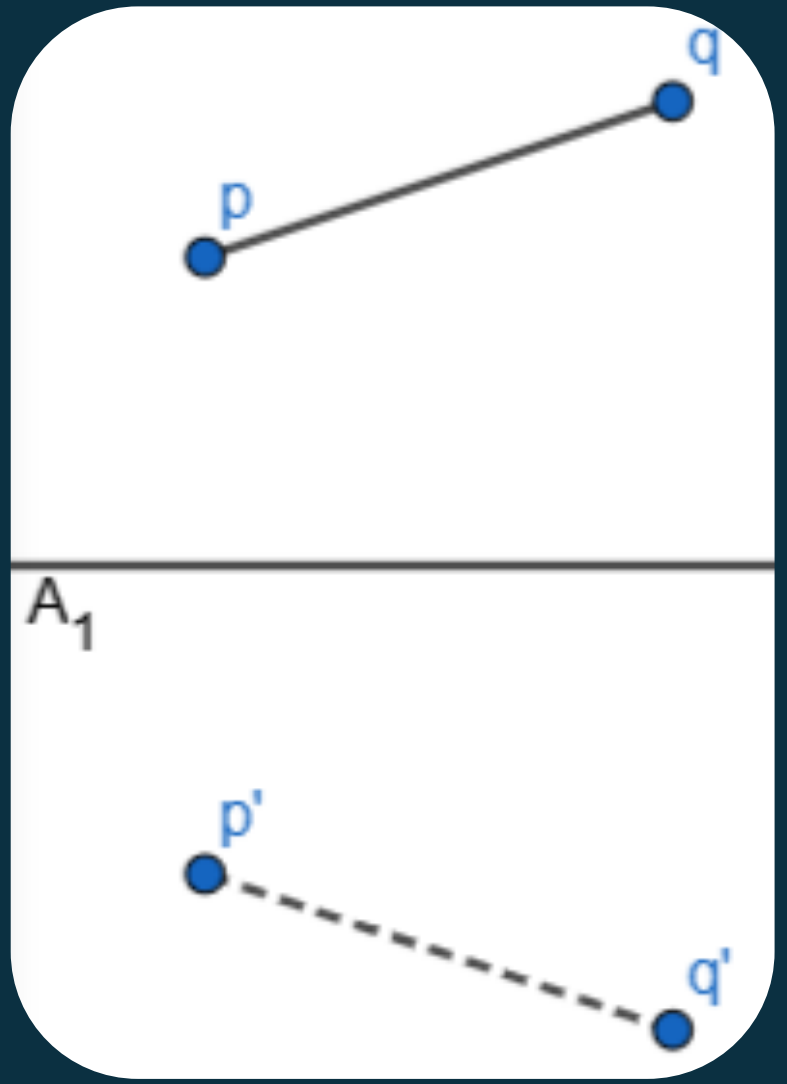
10.5 . $LnA \rightarrow S_A S_A p = p$

10.6 . $LnA \rightarrow \forall p' \exists ! p \; S_A p = p'$



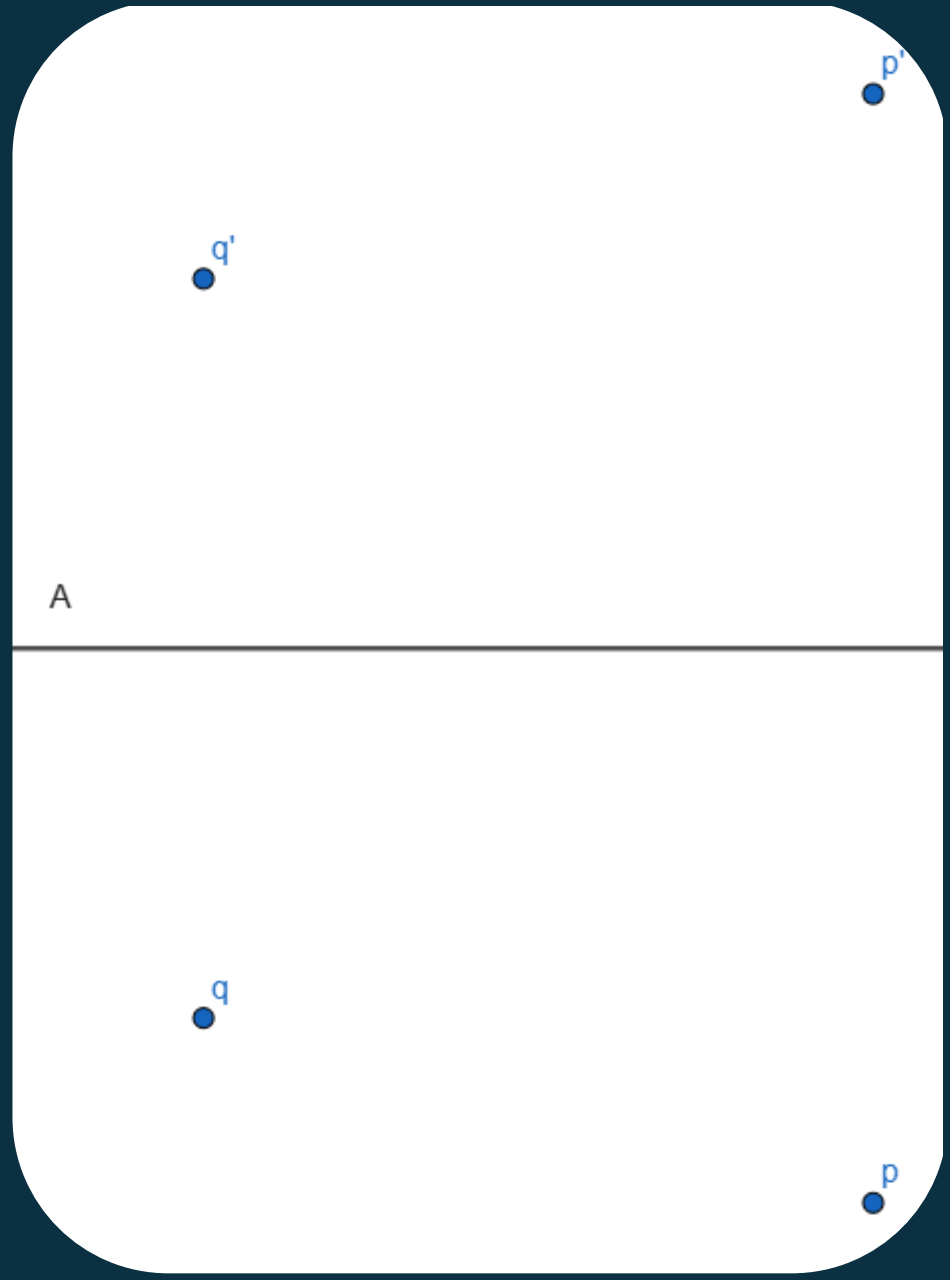
Reflections on lines preserve distances

$10.10.\quad LnA \rightarrow pq \equiv S_A(p)S_A(q)$



Proof of 10.10

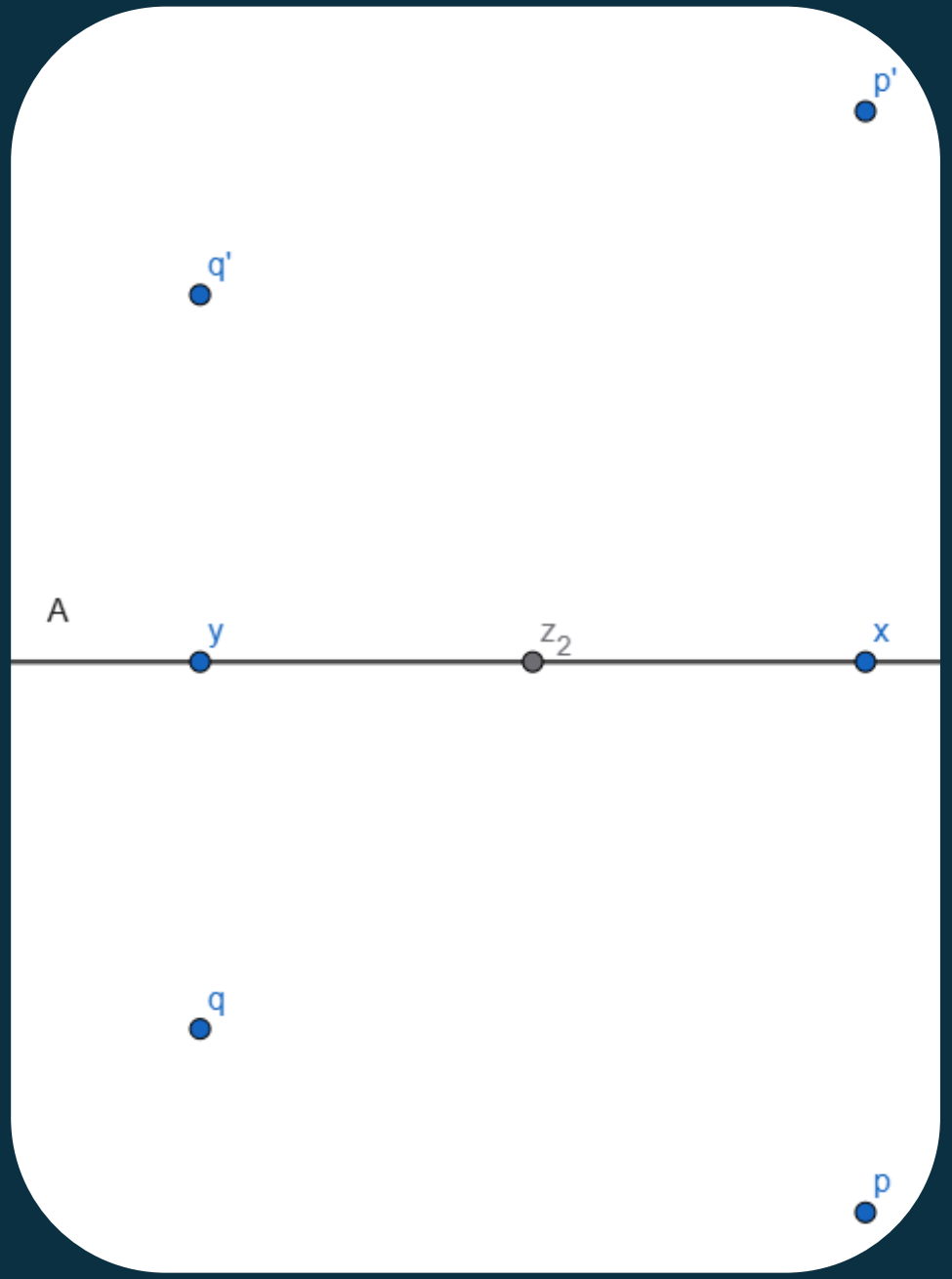
$p' = S_A p$
$q' = S_{Aq}$



Proof of 10.10

$p' = S_A p$
$q' = S_{Aq}$

$x = M(pp')$
$y = M(qq')$
$z = M(xy)$

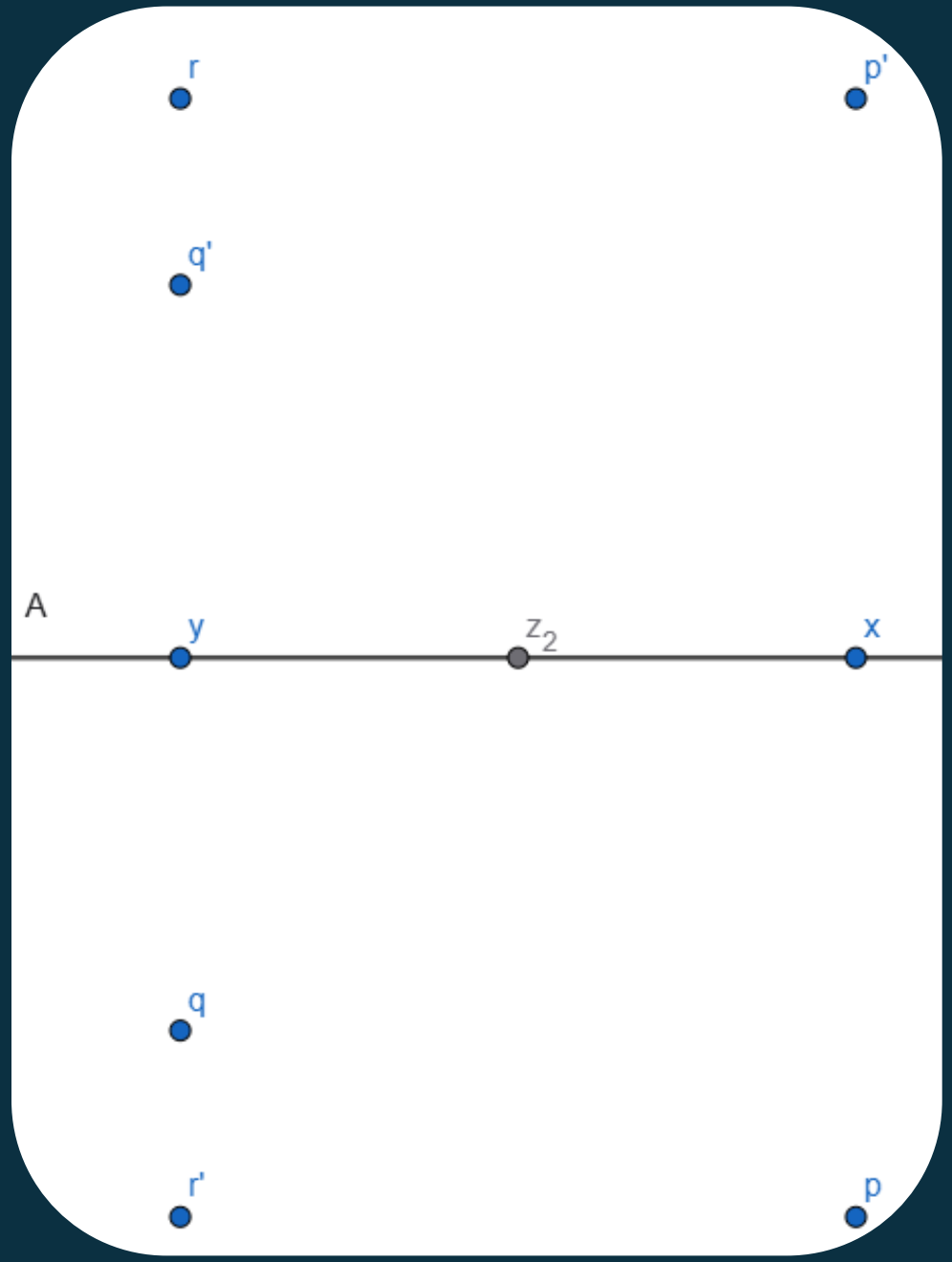


Proof of 10.10

$p' = S_A p$
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$x = M(pp')$
$y = M(qq')$
$z = M(xy)$

$r = S_z(p)$
$r' = S_z(p')$



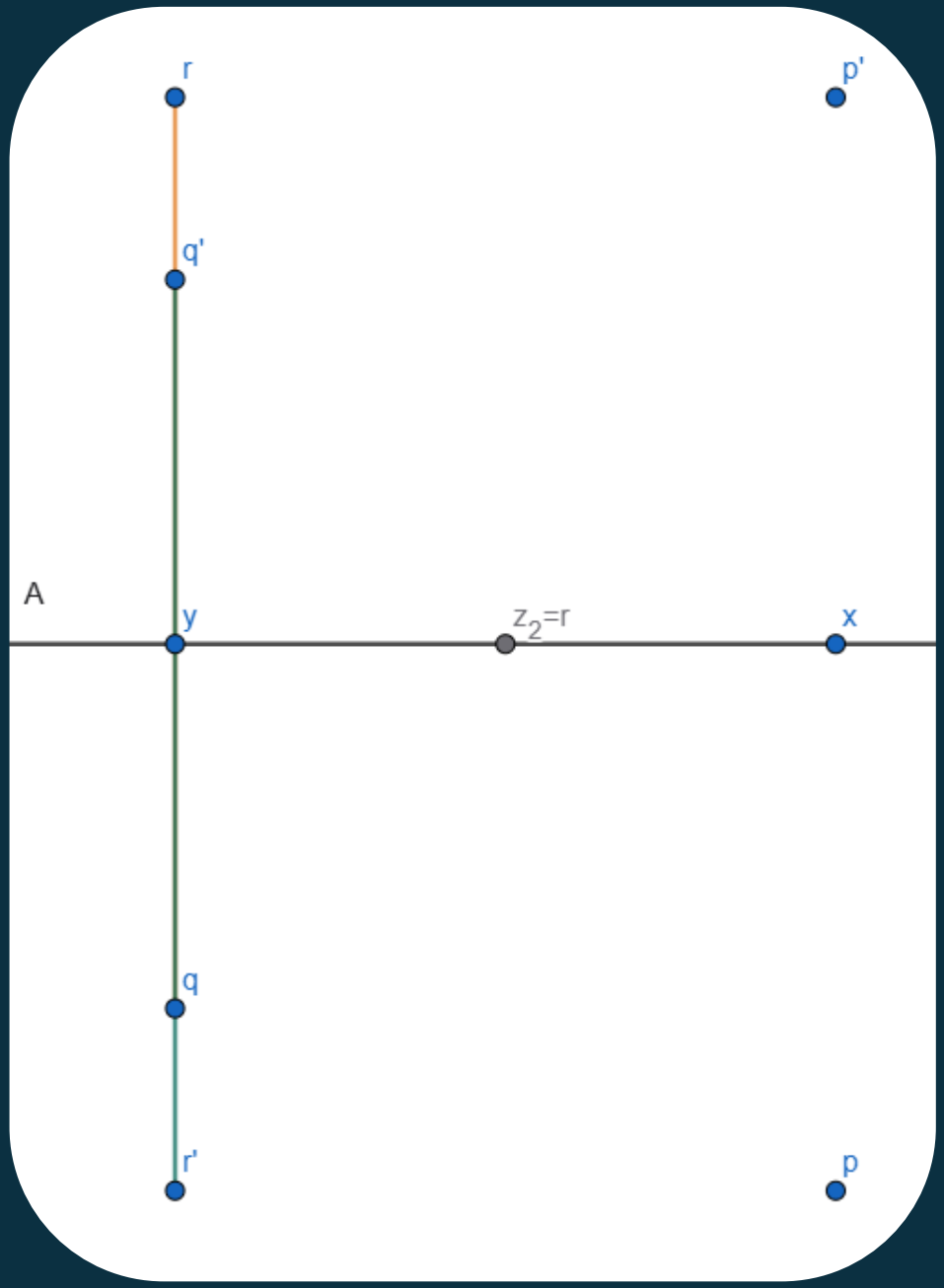
Proof of 10.10

S_z is automorphism (7.14)

- As z=M(xy) and x=M(pp')

→M(rr')=y and so $r' = S_y r$

- 7.13 says: $qr \equiv S_y(q)S_y(r) \equiv q'r'$



Proof of 10.10

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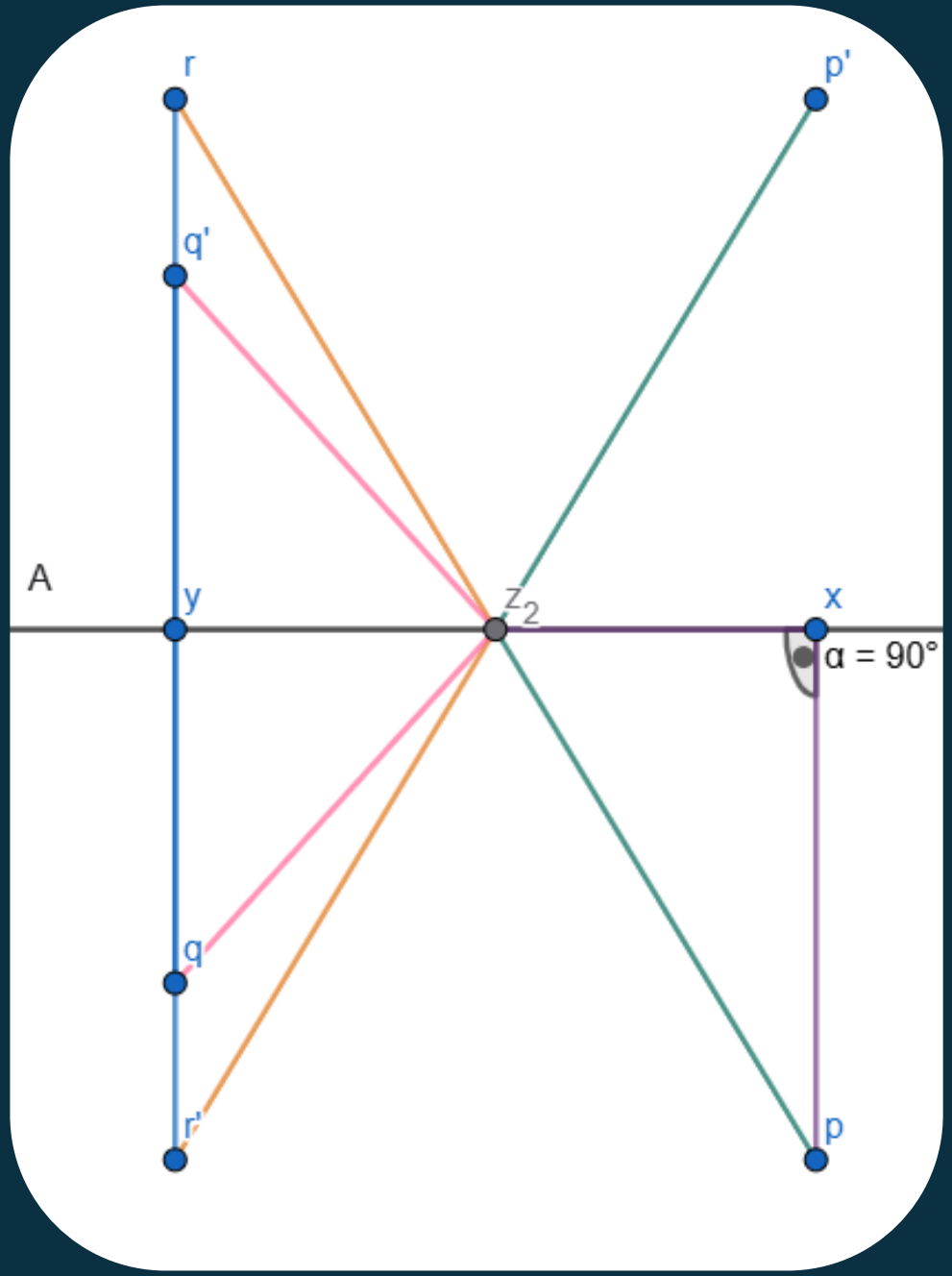
→M(rr')=y and so $r' = S_y r$

- 7.13 says: $qr \equiv S_y(q)\,S_y(r) \equiv q'r'$

- Also: $A \perp px \vee p = x \;\rightarrow Rzxp$

1) zp ≡ zp' and zr ≡ zr'

Analog: 2) zq ≡ zq'



Proof of 10.10

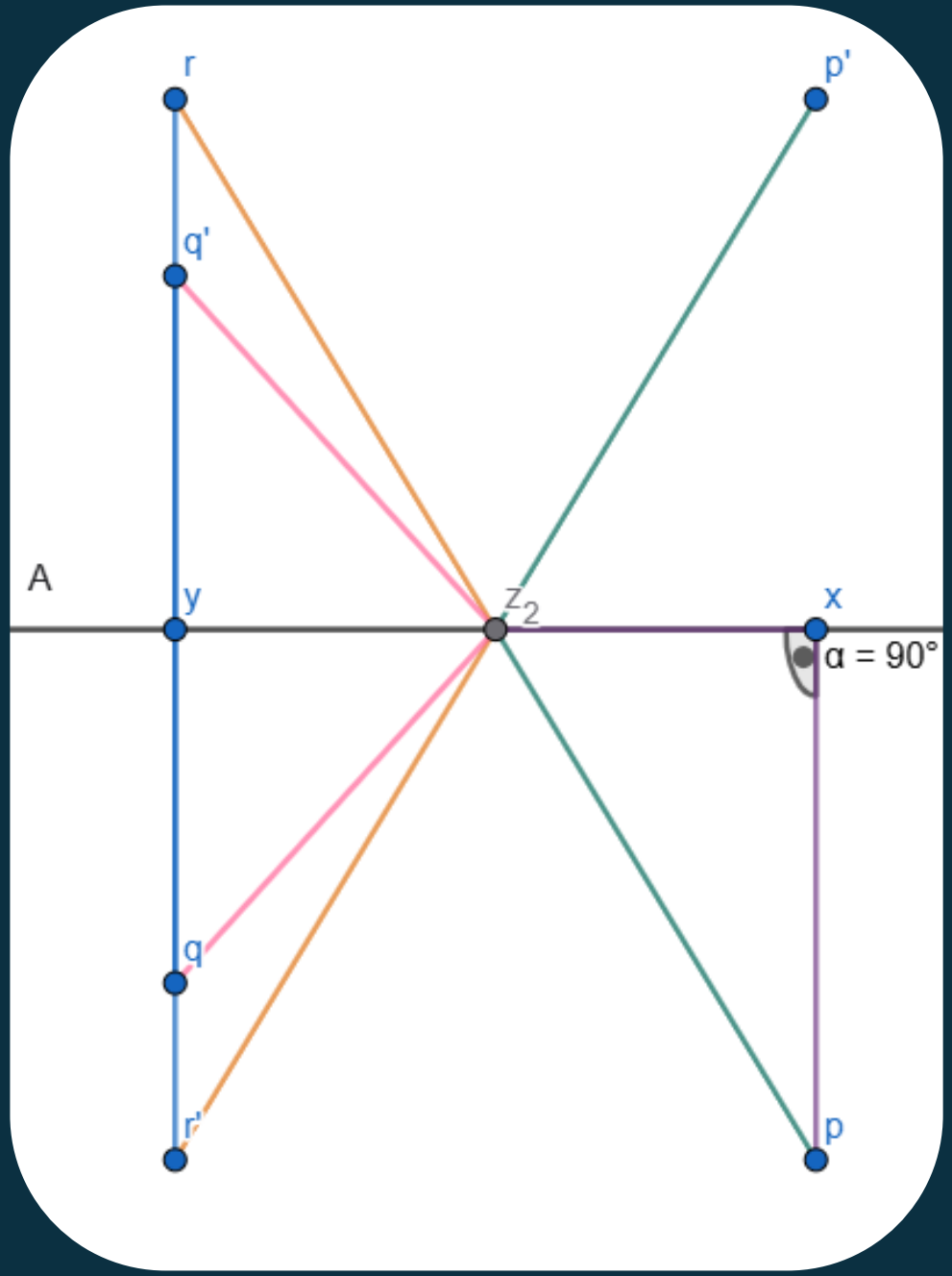
Given:

Brzp and Br’zp’

rq ≡ r’q’

zq ≡ zq’

Also zp ≡ zp’; zr ≡ zr’; pr ≡ p’r’



Proof of 10.10

Given:

Brzp and Br’zp’

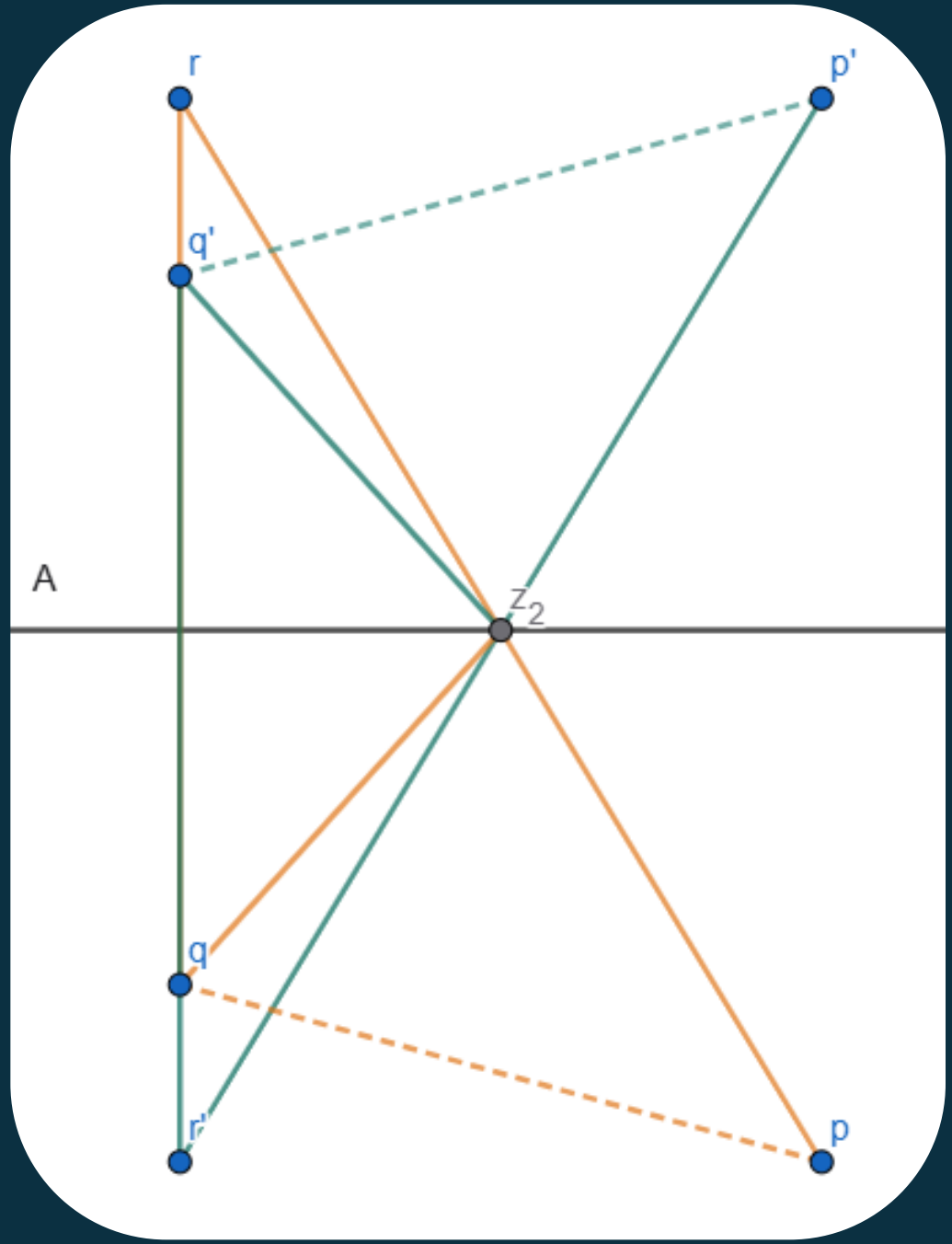
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Use AFS:

$$\begin{pmatrix} rzpq \\ r'zp'q' \end{pmatrix}$$

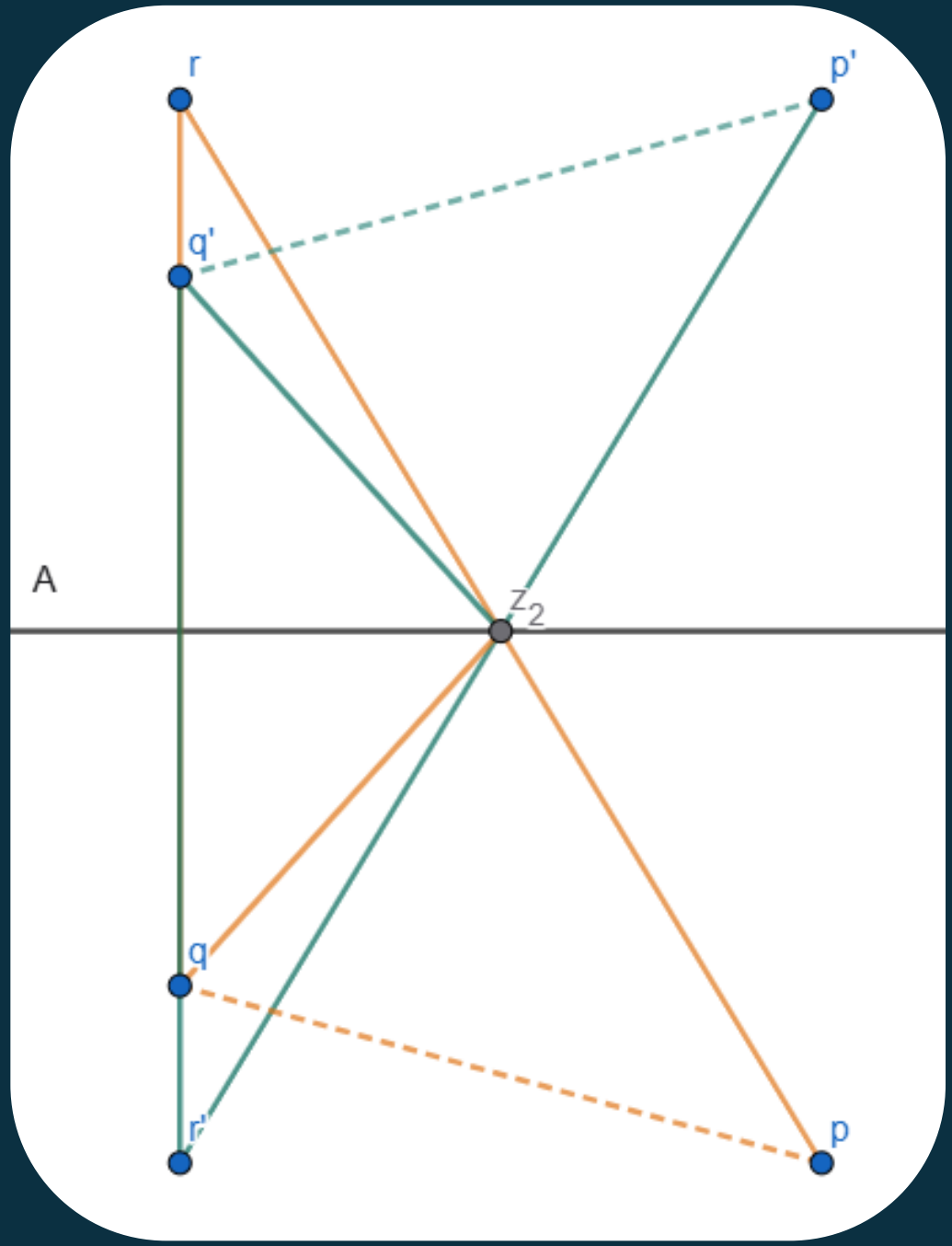


Proof of 10.10

Use AFS:

$$\begin{pmatrix} rzpq \\ r'zp'q' \end{pmatrix}$$

Case 1: $r \neq z \xrightarrow{A5} pq \equiv p'q'$



Proof of 10.10

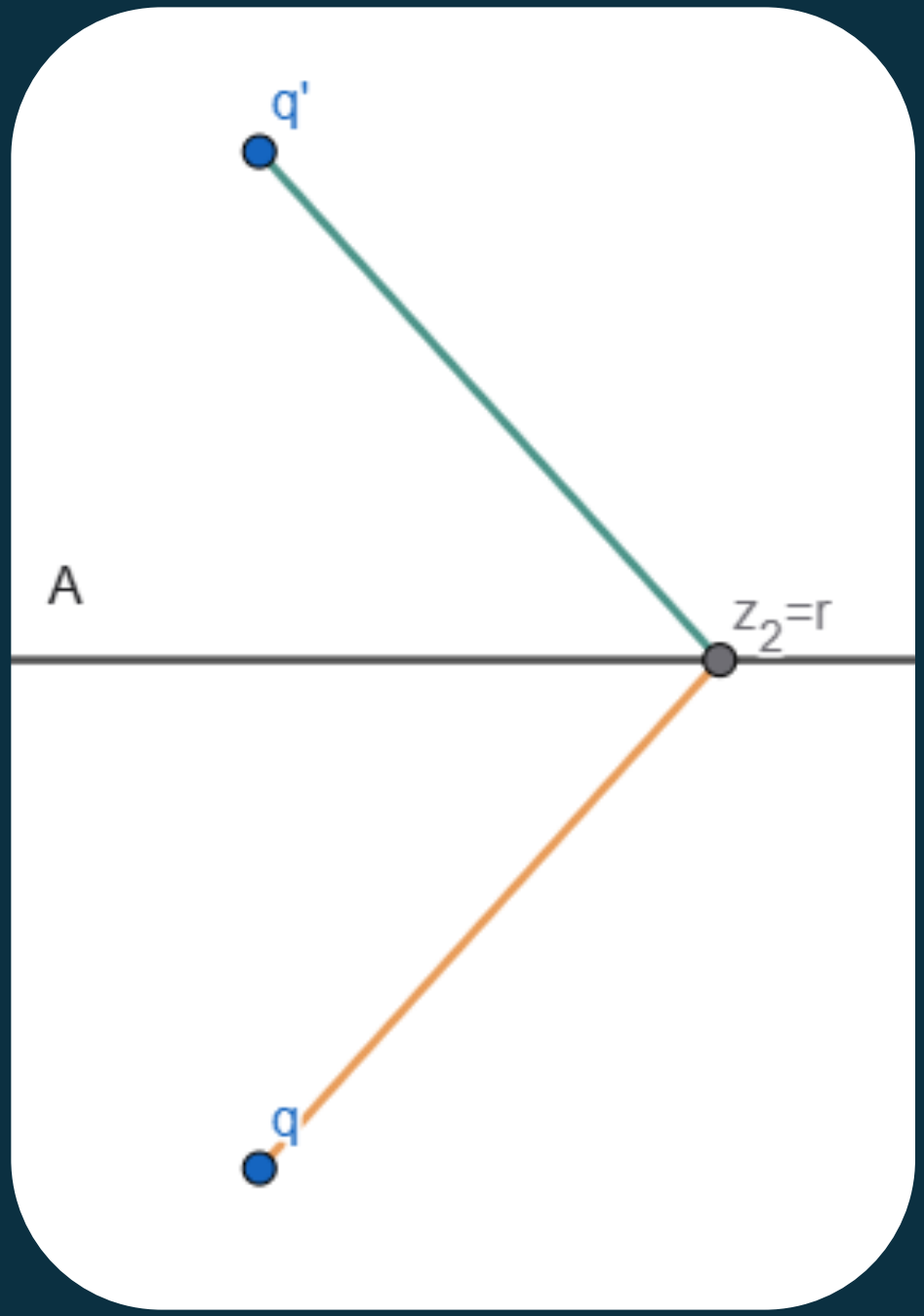
Use AFS:

$$\begin{pmatrix} rzpq \\ r'zp'q' \end{pmatrix}$$

Case 1: $r \neq z \overset{A5}{\rightarrow} pq \equiv p'q'$

Case 2: $r = z \rightarrow z = p = p'$

$$\rightarrow zq \equiv pq \equiv p'q'$$



Thank you for your attention ☺