First results on betweenness and congruence

2.7 Theorem:

The relation ≡ is an equivalence relation between line segments. This means:

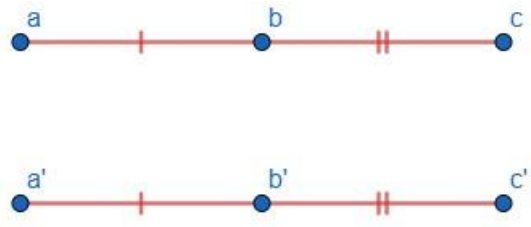
- ab ≡ ba (reflexivity)

- ab ≡ cd → cd ≡ ab (symmetry)

- ab ≡ cd ∧ cd ≡ ef → ab ≡ ef (transitivity)

2.11 theorem (placing line segments end to end):

- Babc ∧ Ba′b′c′ ∧ ab ≡ a′b′ ∧ bc ≡ b′c′ → $ac \equiv a'c'$



3.2 theorem (symmetry of betweenness):

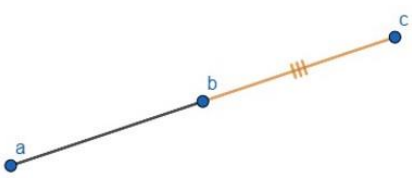
- Babc → Bcba

3.4 Theorem

- Babc ∧ Bbac → a=b

3.14 theorem

- ∃c[Babc ∧ b $\neq c$]



4.1/2 definition/theorem:

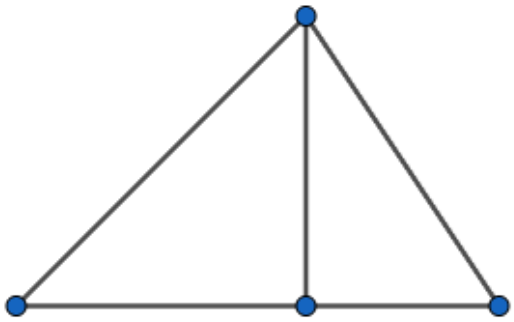
$IFS\begin{pmatrix} a & b & c & d \\ a' & b' & c' & d' \end{pmatrix} :\leftrightarrow$ Babc ∧ Ba′b′c′ ∧ ac ≡ a′c′ ∧ bc ≡ b′c′ ∧ ad ≡ $a'd'$ ∧ cd ≡ $c'd'$

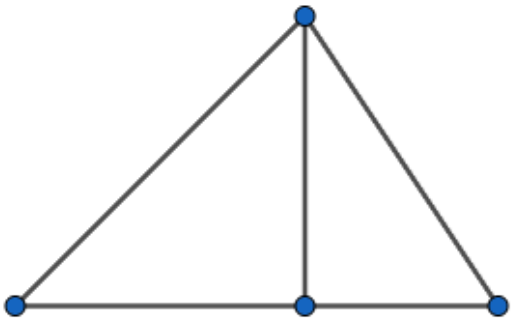
$IFS\begin{pmatrix} a & b & c & d \\ a' & b' & c' & d' \end{pmatrix} \rightarrow bd \equiv b'd$

4.3 theorem:

- Babc ∧ Ba′b′c' ∧ ac ≡ a′c′ ∧ bc ≡ b′c′ → ab ≡ a′b′

Exercise: try to prove 4.3





(hint**:** Try to label the marked points in a way that allows you to effectively use definition/theorem 4.1/2)

4.15 definition (five-segment-configuration):

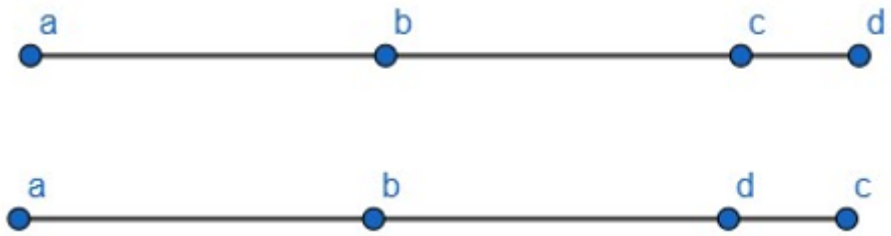
$FS\begin{pmatrix} a & b & c & d \\ a' & b' & c' & d' \end{pmatrix} :\leftrightarrow$ **Col**abc ∧ (abc) ≡ (a′b′c′) ∧ ad ≡ a′d′ ∧ bd ≡ $b'd'$

4.16 theorem:

$FS\begin{pmatrix} a & b & c & d \\ a' & b' & c' & d' \end{pmatrix} \wedge$ a $\neq b \rightarrow cd \equiv$ c′d′

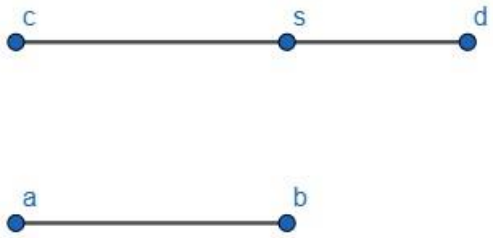
5.1theorem:

- a $\neq b \wedge Babc \wedge Babd \rightarrow Bacd \vee Badc$



5.4 definition:

- ab $\leq cd :\leftrightarrow \exists s[B$csd $\wedge$ ab $\equiv cs]$



- cd $\geq ab :\leftrightarrow ab \leq cd$

5.6 theorem:

ab $\leq$ cd $\wedge$ ab $\equiv$ a′b′ $\wedge$ cd $\equiv$ c′d′ $\rightarrow$ a′b′ $\leq$ c′d′

5.8 theorem:

ab $\leq$ cd $\wedge$ cd $\leq$ ef $\rightarrow$ ab $\leq$ ef

5.9 theorem:

ab $\leq$ cd $\wedge$ cd $\leq$ ab $\rightarrow$ ab $\equiv$ cd