# HEGL Illustrating Mathematics Seminar
## Winter Semester 2024-2025

$$\boxed{\lambda\Pi\times\Sigma}$$

## Logic, algebra and proof visualization

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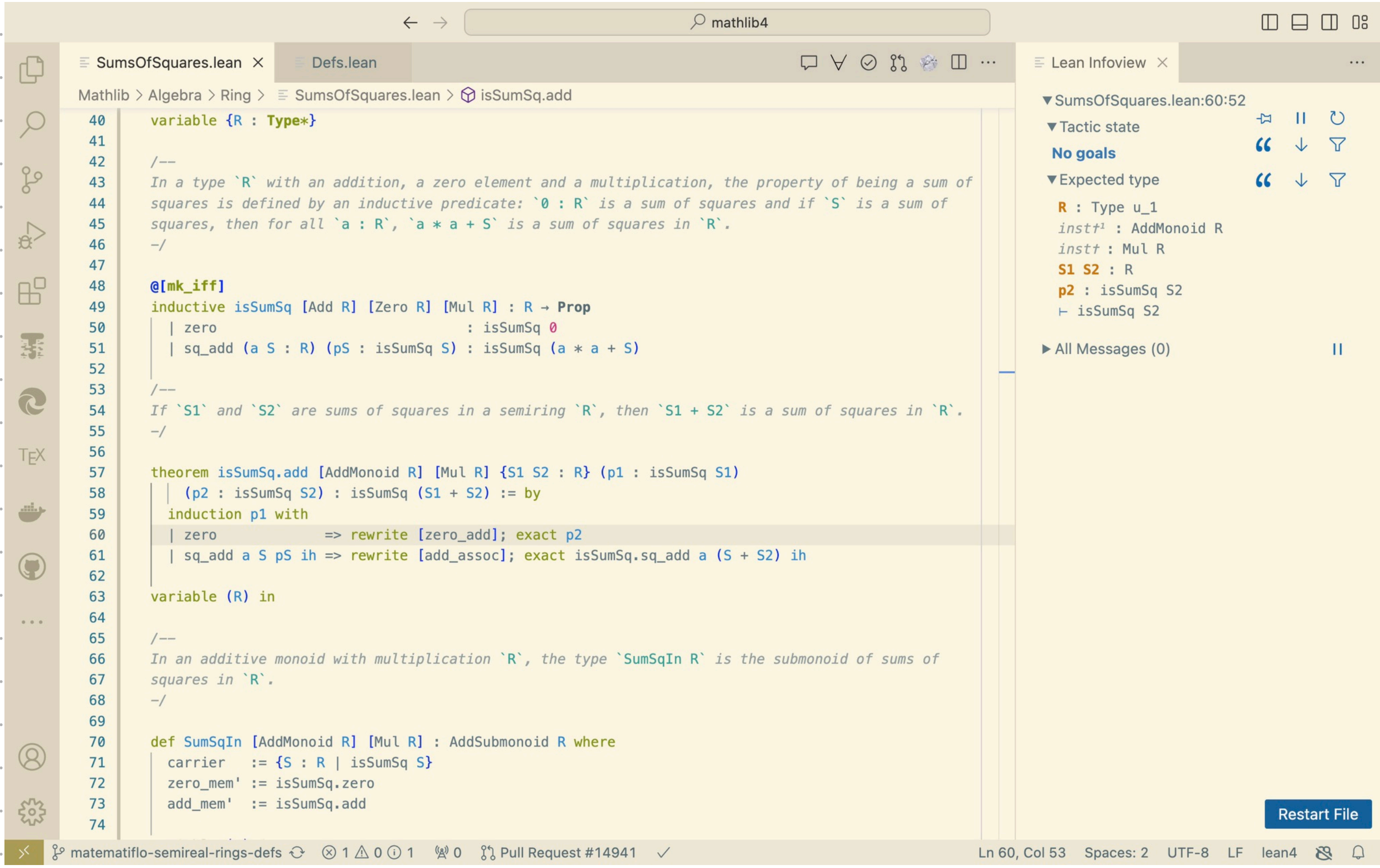
## What is a mathematical proof and how can we represent it?

- In this seminar, we take a constructive approach to this question.
- Proving a theorem means constructing a proof of it, much like when you introduce a term of a certain type in a programming language.
- The underlying logical foundations of this approach make functions the primitive object, from which everything else is defined.

Can we use this to venture a guess as to how mathematics will be taught at the university level 50 years from now?

Programming Language and Theorem Prover



## Constructive mathematics

The constructive approach to mathematics is often reduced (quite misleadingly) to mathematics *without* the law of excluded middle (LEM), or without the axiom of choice (AC). It should instead be thought of as a sum of algorithmic procedures for deriving proofs, which *may or may not* use LEM or AC.

Conceptually, this is made possible by an interpretation of mathematical statements in a way that lays the emphasis on computation. Existential statements, in particular, should be formally interpreted in a constructive manner (which calls for an explicit construction of an element satisfying a certain property).

Additionally, logic is not a pre-requisite in this approach. Instead, logical connectives and quantifiers are introduced formally in constructive mathematics, using the same language used to introduce mathematical objects.

## List of topics for the talks

- Sets and logic.
- Basic algebra.
- Rings and modules.
- Divisibility in discrete domains.
- Principal ideal domains.

In the seminar, we will focus on undergraduate commutative algebra: rings, fields and algebras over a field.

As a first step, participants will give a presentation on a subject of their choice from the list of topics proposed by the coordinator.

This talk, and its corresponding handout, will count towards 50% of the final grade.

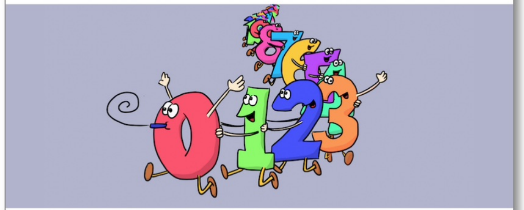
In the second step, we will work collaboratively to elaborate an online tutorial to some of the mathematical results presented in the first part.

# Lean Game Server

A repository of learning games for the proof assistant Lean *(Lean 4)* and its mathematical library mathlib

### Natural Number Game

The classical introduction game for Lean.

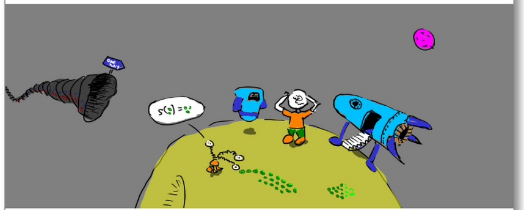


In this game you recreate the natural numbers ℕ from the Peano axioms, learning the basics about theorem proving in Lean.

This is a good first introduction to Lean!

### Robo

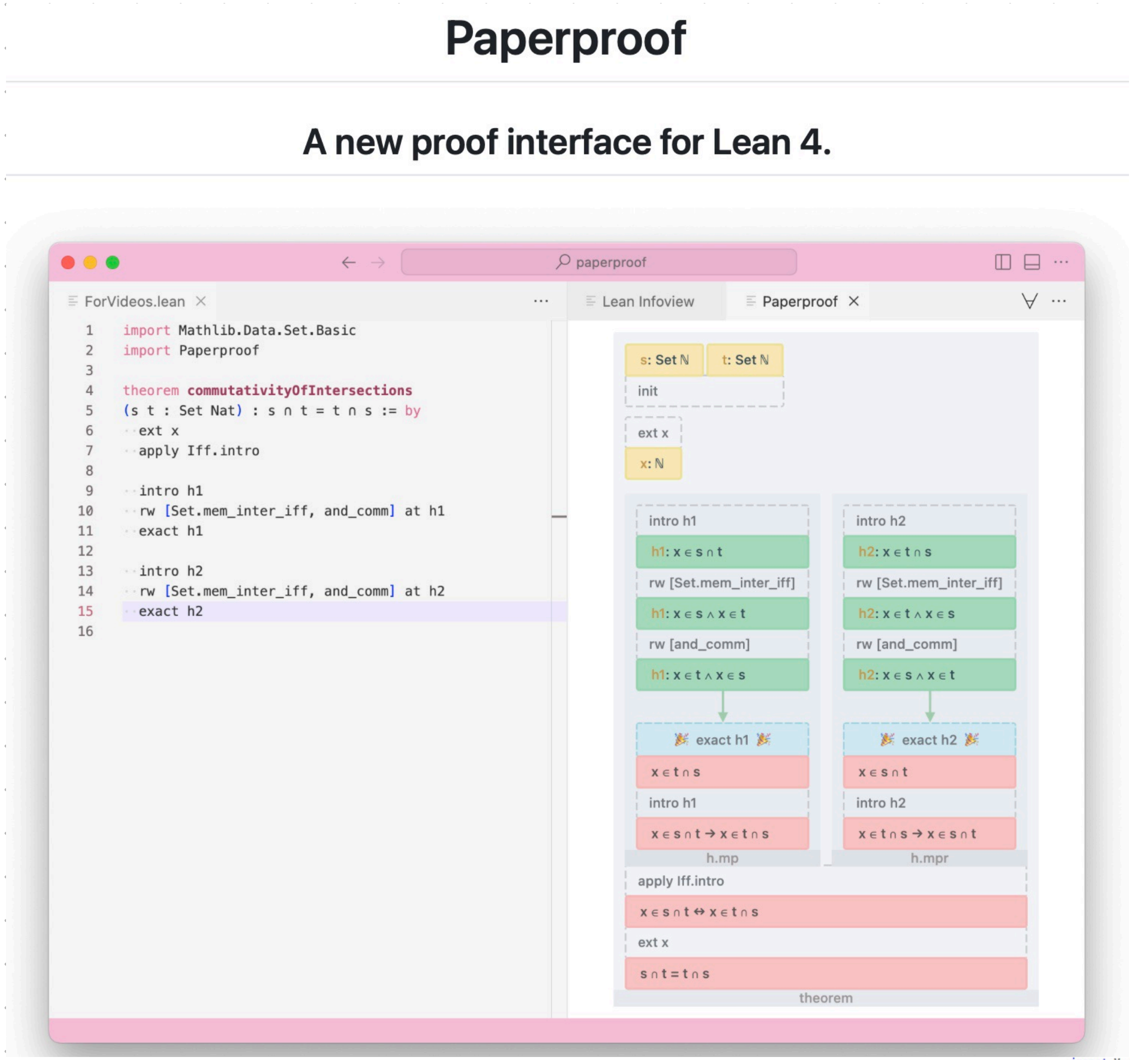
Erkunde das Leansche Universum mit deinem Robo, welcher dir bei der Verständigung mit den Formalosophen zur Seite steht.



Dieses Spiel führt die Grundlagen zur Beweisführung in Lean ein und schneidet danach verschiedene Bereiche des Bachelorstudiums an.

(Das Spiel befindet sich noch in der Entstehungsphase.)

The ideal output is a kind of "Heidelberg Lean game" that can be played online by future generations of students, to teach themselves the basics of type-theoretic mathematics.




Other proof visualization techniques such as Paperproof or Verbose might be explored too.

The final presentation and HEGL blog post, by the students, will count towards the remaining 50% of the grade.